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Lovick Edward Cannon, III

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13. ABSTRACT

Pseudo random number generating programs coded in machine language for the IBM 360/65 computer have been prepared for producing random values from the normal, exponential, and gamma distributions as well as from any discrete probability distribution. The schemes are based largely on combinations of sophisticated techniques first suggested by Marsaglia. The distributions simulated are exact within the word size of the computer and average production time per number is very short, e.g. 10 to 40 microseconds depending upon the particular distribution.

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LOVICK EDWARD CANNON, III

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ROLF BARGMANN
PRINCIPAL INVESTIGATOR

THE UNIVERSITY OF GEORGIA
DEPARTMENT OF STATISTICS AND COMPUTER SCIENCES
ATHENS, GEORGIA 30601

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CHAPTER 1
INTRODUCTION

With the increasing use of electronic computers, random sampling methods have become a very useful tool for providing solutions to problems involving probability as well as for problems of a deterministic nature. The availability of sequences of numbers which appear to be drawn at random from particular probability distributions is a vital ingredient in the random sampling process. Such numbers are referred to as pseudo-random numbers or, for convenience, simply as random numbers. This paper is concerned with the rapid and accurate generation of such numbers in a stored computer program.

The use of random sampling to estimate distribution functions originated with Student [17] in 1908. At that time and for some time to follow, necessary random numbers were obtained by drawing cards from a deck, counters from an urn, or by rolling dice. Such processes are very slow and make it quite difficult to insure randomness.

To facilitate the use of random numbers large tables of random digits were compiled. The first such table, published by Tippett [19] in 1927, consisted of 41,600 digits taken at random from census reports and combined into 10,400 four digit numbers. The requirement for a larger set of numbers led to the publication of 100,000 random digits in 1939 by Kendall and Babington-Smith [7]. These digits were generated by means of

a mechanical process and were the first to be so produced. Kendall and Babington-Smith are also responsible for developing many of the tests frequently applied to sequences of random numbers (5,6). Other such tables have since been constructed, primarily by the use of physical devices, culminating in the most extensive, "A Million Random Digits with 100,000 Normal Deviates" published by the RAND Corporation [15] in 1955.

The use of a physical device for the generation of random numbers on line with a computer is both expensive and difficult to maintain. The storing of a table of random numbers on magnetic tape or on cards is also an unsatisfactory method of generating numbers for use in a computer. This would necessitate the use of an input device; also the time required to read in the numbers would be excessive.

The development of arithmetic procedures for the generation of random numbers began in the 1940's with the introduction of computers. The first such procedure was suggested by von Neumann and Metropolis in 1946 and is described in [4]. It was also during this time that calculations involving random numbers received the picturesque name "Monte Carlo." The "middle-square" method proposed by von Neumann and Metropolis is simple, fast, and requires only an initial starting value. In this procedure each new number is produced by taking the middle n digits of the square of the previous n-digit number. As pointed out in [4] and in [16] the sequence of numbers produced using this method sooner or later degenerates to a cycle which often is very small, and at worst consists of only a single number. In addition some of the statistical tests performed on samples generated by the middle-square procedure have resulted in failures. Improvements on the von Neumann method are plagued by similar difficulties

[2] and [20].

Of the various arithmetic procedures sequences of random numbers with the best statistical properties and longest periods are generated by means of the congruence relation

$$x_{n+1} = a \cdot x_n + c \cdot (\text{mod } m). \tag{1.1}$$

The representation (1.1) is termed the mixed congruential method. The multiplicative congruential method is defined by taking c = 0 in (1.1) Lehmer [8] is credited with the invention of the multiplicative method. In their very informative survey Hull and Dobell [3] prescribe conditions for a, m, c, and x_0 which insure maximum period. Other procedures for the genration of random numbers based upon reduced Fibonacci series [21] and upon transcendental numbers [3] are inferior to those based on the congruence relation (1.1) (See [4]). In [9] MacLaren and Marsaglia present a table look-up scheme which seems to offer some promise.

Random numbers generated by arithmetic procedures are not "truly random" in that the entire sequence is determined in advance and can be reproduced simply by using the same starting value, x_0. The statistical behavior of sequences generated by both the mixed and multiplicative congruential methods is quite good, and, in fact, numbers genrated in this fashion do appear to be drawn at random from the uniform (0,1) distribution

$$f(u) = 1, \; 0<u<1. \tag{1.2}$$

The reproducibility of sequences is an advantage in debugging and in certain calculations it may be desirable to reproduce a given sequence.

Bargmann [1] has provided a procedure for the generation of independent uniform (0,1) random numbers on a binary computer. This procedure, which is a form of the multiplicative congruential method, is defined by

$$u_{n+1} = a \cdot u_n (\text{mod } 2^{32}) \qquad (1.3)$$

where a is chosen such that a + 1 and a - 1 end in as few zero bits as possible. This is insured by requiring that neither a - 1 nor a + 1 be divisible by 2^k for k = 3,4,.... Choosing a as an odd power of 5 determines that the low order 3 digits will be 125, and neither 124 nor 126 is divisible by 2^k for k = 3,4,.... Hence, $a = 5^{13}$ is a reasonable choice. This procedure requires that u_0, the starting value, be an odd positive integer.

The use of this procedure results in a very fast computer program requiring only 3 operations: load, multiply, and store. A description of the usage of this program along with time and storage requirements is given in Chapter 6 of this paper. Results of statistical tests performed on this procedure are presented in Chapter 5, and a listing of the program is given in Appendix A.

The preceding discussion has dealt only with the generation of random numbers which appear to be uniformly distributed. In principle it should be very easy to obtain any other distribution from the uniform distribution, requiring only a solution to the equation

$$u = F(y) \qquad (1.4)$$

for y, where u is uniformly distributed on the interval (0,1) and F is the

required cumulative distribution function. When the inverse of F is known, as for the exponential distribution, this is a simple matter:

$$y = -\ln(u). \tag{1.5}$$

However, the evaluation of the ln function may be somewhat time consuming. An alternative approach might be to store a value of y for each possible value of u based on the relation

$$y = F^{-1}(u). \tag{1.6}$$

To generate y, simply generate a uniform (0,1) random number u which will determine the location of a stored value of y. Let u be given to 8 digits and let β be the base of the number system in which the digits are represented, then for this procedure a total of β^8 storage locations are required.

Various approximations have been proposed for the normal distribution. Most of these involve taking the sum of a fixed number of uniform (0,1) deviates or the use of Chebyshev approximations [14] or a combination of the two [18]. Such approximations frequently lack accuracy and are either slow or space consuming.

The procedures suggested by Marsaglia et al. in [11],[12], and [13] for transforming uniform (0,1) random numbers to random numbers having other distributions are superior to any encountered. They are fast, require minimal space, and are simple to program. In addition these procedures are completely accurate; the precision of the result is dependent only on the word size of the computer. These procedures along with programs and results of statistical tests are presented in detail in this paper.

CHAPTER 2

GENERATION OF DISCRETE RANDOM VARIABLES IN A COMPUTER

Let Y be a discrete random variable with point probabilities $p_i = Pr(Y = y_i)$ for $i = 1,2,\ldots$. The direct way to generate Y in a computer is to generate a uniform (0,1) random number u and put $Y = y_i$ if $p_1 + p_2 + \ldots + p_{i-1} < u \leq p_1 + p_2 + \ldots + p_i$. However, techniques based on this method lead to complicated programs that are excessively time consuming.

An alternative method proposed by Marsaglia [11] is simple to program and requires minimal time and storage. Let p_i for $i = 1,2,\ldots,n$ be expressed by k digits as $p_i = .\delta_{1i}\delta_{2i} \ldots \delta_{ki}$. If the domain of the random variable is infinite, the probability distribution must be truncated at some p_n. The fastest method for generating Y is as follows.

Let β be the base of the number system in which the δ_{ji}'s are represented. In memory locations 0 to (β^k-1) store $\delta_{11}\delta_{21}\delta_{31} \ldots \delta_{k1}$ y_1's, $\delta_{12}\delta_{22} \ldots \delta_{k2}$ y_2's, ..., $\delta_{1n}\delta_{2n} \ldots \delta_{kn}$ y_n's. If u is a uniform (0,1) random number, $u = .d_1d_2 \ldots d_k$, look up the number in location $d_1d_2 \ldots d_k$ and let that be Y.

Though this may be the fastest method, it clearly requires an exessive amount of storage space. Even if the p_i's are truncated to four digits, β^4 memory locations will be required.

Marsaglia [11] suggests a technique that offers a considerable reduction in storage space with very little sacrifice in execution time. For convenience assume that the p_i's are truncated to four digits. We have the following situation:

Value of Y	Probability
y_1	$.\delta_{11}\delta_{21}\delta_{31}\delta_{41}$
y_2	$.\delta_{12}\delta_{22}\delta_{32}\delta_{42}$
y_n	$.\delta_{1n}\delta_{2n}\delta_{3n}\delta_{4n}$

Define

$$P_0 = 0, \qquad P_r = \beta^{-r} \sum_{i=1}^{n} \delta_{ri} \tag{2.0.1}$$

for r = 1,2,3,4, and

$$\Pi_s = \sum_{j=1}^{s} \sum_{i=1}^{n} \delta_{ji} \tag{2.0.2}$$

for s = 1,2,3,4 and $\Pi_0 = 0$. Segment memory locations 0 to $(\Pi_4 - 1)$ into four mutually exclusive sets such that set 1 consists of locations 0 to $(\Pi_1 - 1)$, set 2 comprises locations Π_1 to $(\Pi_2 - 1)$, and in general the s^{th} set occupies memory locations Π_{s-1} to $(\Pi_s - 1)$ for s = 1,2,3,4. In each set s for s = 1,2,3,4 each y_i is stored in δ_{si} locations for i = 1,2, ..., n, a total of $\Pi_s - \Pi_{s-1}$ locations. The total memory requirement is then $\Pi_4 = \sum_{j=1}^{4} \sum_{i=1}^{n} \delta_{ji}$ locations. Now choose set 1 with probability P_1, set 2 with probability P_2, set 3 with probability P_3, or

set 4 with probability P_4. Having chosen a set, select at random a location within that set; the number occupying that location is the desired random variable.

This procedure gives the required discrete distribution, as can be seen by defining:

$$q_{ri} = \delta_{ri} / \sum_{i=1}^{n} \delta_{ri}$$

for $i = 1,2, \ldots, n$ and $r = 1,2,3,4$. Thus q_{ri} is simply the probability of choosing y_i from set r. Then the probability of generating $Y = y_i$ is

$$\sum_{r=1}^{4} P_r \cdot q_{ri} = \sum_{r=1}^{4} \beta^{-r}\delta_{ri} = .\delta_{1i}\delta_{2i}\delta_{3i}\delta_{4i} \quad ,$$

which is the probability $P(Y = y_i)$.

In order to select the proper set and location within that set, generate a uniform (0,1) random variable $u = .d_1d_2\ldots$, and let $A\{j\}$ denote the contents of memory location j. Then if

$$\sum_{r=0}^{s-1} P_r \leq u \leq \sum_{r=0}^{s} P_r$$

put

$$Y = A\ \{d_1d_2 \ldots d_s + \Pi_{s-1} - \beta^s \sum_{r=0}^{s-1} P_r\}.$$

The following example illustrates this procedure. The hexadecimal (β=16) number system is used as it is the representation of the IBM 360/65,

i	y_i	p_i
1	0	0.A000
2	1	0.3800
3	2	0.14A2
4	3	0.0F50
5	4	0.0202
6	5	0.020C

Defining P_r and Π_s as before gives

$$P_0 = 0$$

$$P_1 = 16^{-1} \sum_{i=1}^{6} \delta_{1i} = 0.E$$

$$P_2 = 16^{-2} \sum_{i=1}^{6} \delta_{2i} = 0.1F$$

$$P_3 = 16^{-3} \sum_{i=1}^{6} \delta_{3i} = 0.00F$$

$$P_4 = 16^{-4} \sum_{i=1}^{6} \delta_{4i} = 0.0010$$

and

$$\Pi_0 = 0$$

$$\Pi_1 = \sum_{j=1}^{1} \sum_{i=1}^{6} \delta_{ji} = E$$

$$\Pi_2 = \sum_{j=1}^{2} \sum_{i=1}^{6} \delta_{ji} = 2D$$

$$\Pi_3 = \sum_{j=1}^{3} \sum_{i=1}^{6} \delta_{ji} = 3C$$

$$\Pi_4 = \sum_{j=1}^{4} \sum_{i=1}^{6} \delta_{ji} = 4C.$$

The four sets are stored as follows:

A - TABLE

SET 1		SET 2				SET 3		SET 4	
Loc.	Con.	Loc.	Con.	Loc.	Con.	Loc.	Con.	Loc.	Con.
0	0	E	1	1E	3	2D	2	3C	2
1	0	F	1	1F	3	2E	2	3D	2
2	0	10	1	20	3	2F	2	3E	4
3	0	11	1	21	3	30	2	3F	4
4	0	12	1	22	3	31	2	40	5
5	0	13	1	23	3	32	2	41	5
6	0	14	1	24	3	33	2	42	5
7	0	15	1	25	3	34	2	43	5
8	0	16	2	26	3	35	2	44	5
9	0	17	2	27	3	36	2	45	5
A	1	18	2	28	3	37	3	46	5
B	1	19	2	29	4	38	3	47	5
C	1	1A	3	2A	4	39	3	48	5
D	2	1B	3	2B	5	3A	3	49	5
		1C	3	2C	5	3B	3	4A	5
		1D	3					4B	5

TABLE 2.1

To generate the desired random variable, let $u = .d_1d_2 \ldots$ be a uniform (0,1) random number and let $A\{j\}$ denote the contents of memory location j and proceed as follows:

1) If $0 \leq u < 0.E$ put $Y = A\{d_1\}$

2) If $0.E0 \leq u < 0.FF$ put $Y = A\{d_1d_2 - D2\}$

3) If $0.FF0 \leq u < 0.FFF$ put $Y = A\{d_1d_2d_3 - FC3\}$

4) If $0.FFF0 \leq u$ put $Y = A\{d_1d_2d_3d_4 - FFB4\}$.

Examples:

$u = 0.2170 \ldots$ $Y = A\{2\} = 0$

$u = 0.EF10 \ldots$ $Y = A\{EF\text{-}D2\} = A\{1D\} = 3$

$u = 0.FFFE \ldots$ $Y = A\{FFFE - FFB4\} = A\{4A\} = 5$.

In this example $4C_{(16)}$ memory locations are required as compared to 16^4 memory locations for the fastest method. Suppose the times for certain operations in the computer are:

Operation	Time
Compare two integers	P
Subtract two integers	S
Look up an addressed location	L

Then the fastest method for generating Y requires a total time of P + L and 16^4 memory locations. In the example presented only $4C_{(16)}$ memory locations are required and an average generation time of

$$0.E \cdot (P + L) + 0.F \cdot (2P + S + L) + 0.00F \cdot (3P + S + L) + 0.001 \cdot (3P + S + L) = 1.21_{(16)} \cdot P + 0.20_{(16)} \cdot S + L$$

A schemacic for storing the four sets of discrete variables and the subsequent generation of the desired random variables is presented in figure 2.1.

FIGURE 2.1

Flowchart for the Generation of Random Numbers from a Discrete Distribution

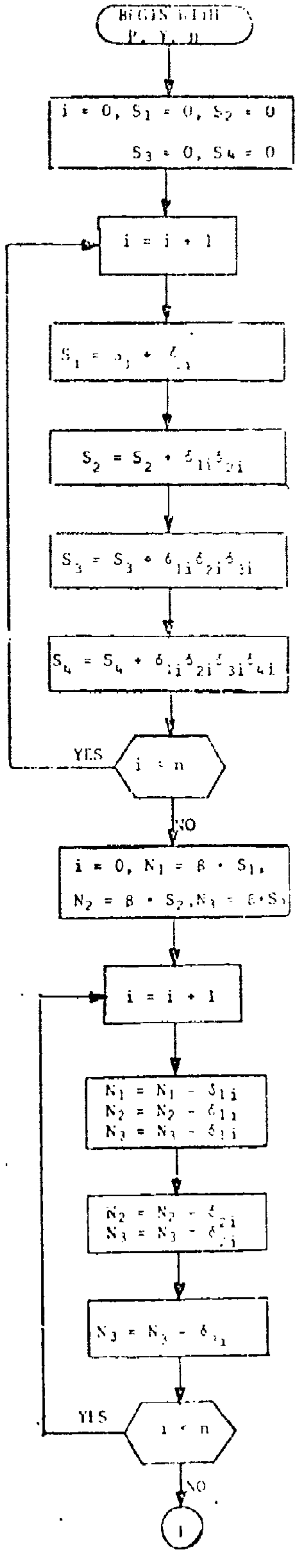
P is a vector of probabilities ($p_i = .\delta_{1i}\delta_{2i}\delta_{3i}\ldots$)

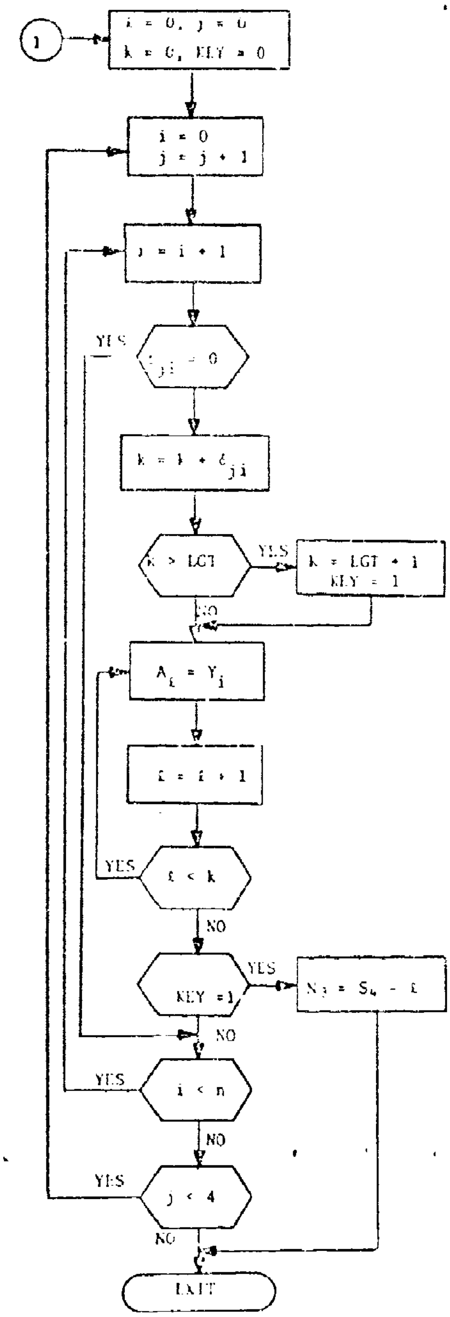
Y is a vector of the corresponding discrete variables (integer mode)

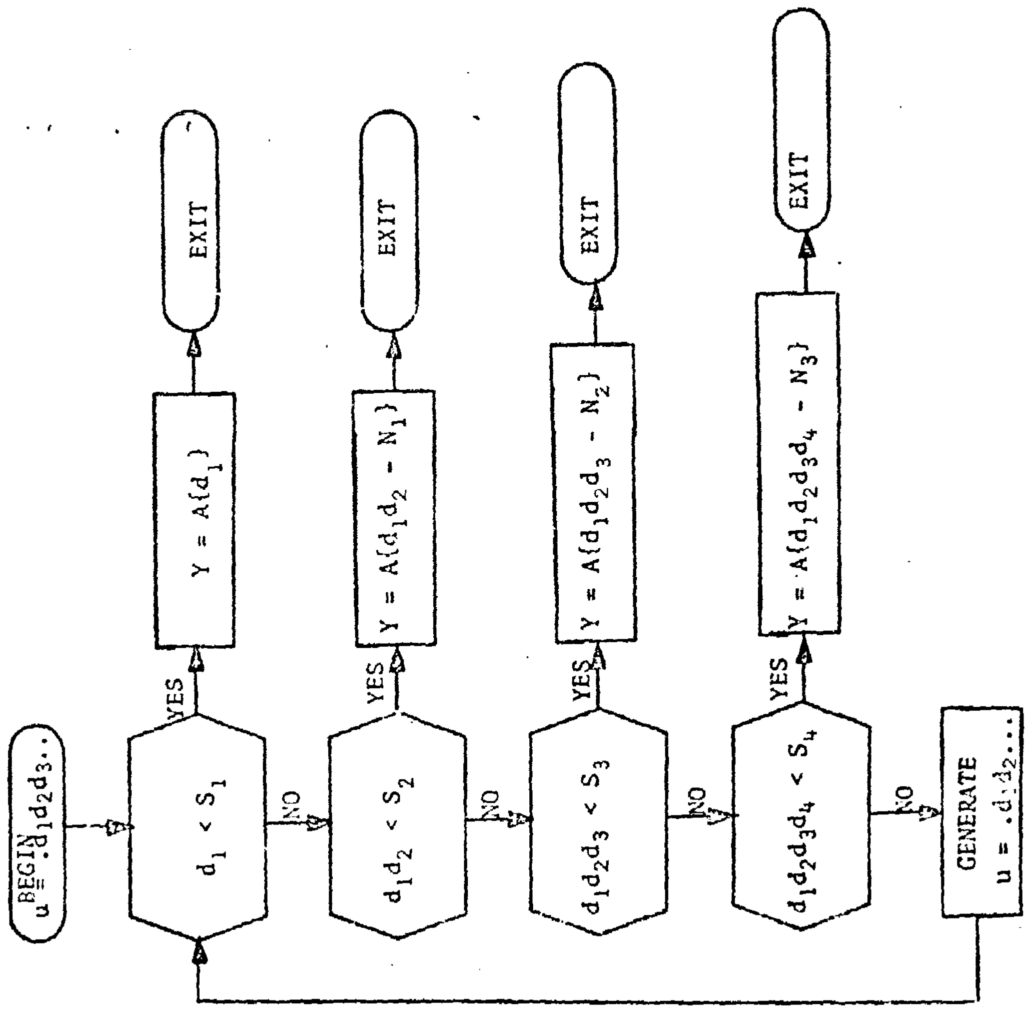
n is the number of elements in P and in Y.

β is the base of the number system in which the δ_{ij}'s are represented (for the 360/65 β = 16, for a decimal machine β = 10).

LGT is the maximum length of the A-Table. If LGT is exceeded, the table loading process ceases and N_3 is adjusted (since N_3 was calculated on the basis that loading would be complete).


BEGIN WITH
P, Y, n
j = 0, S1 = 0, S2 = 0
S3 = 0, S4 = 0
i = i + 1
YES
i < n
NO
i = i + 1
YES
NO
1


1
i = 0
j = j + 1
YES
k > LGT
YES
NO
KEY = 1
YES
NO
i < n
YES
NO
j < 4
YES
NO
EXIT


BEGIN
u = .d1d2d3...
d1 < S1
YES
Y = A{d1}
EXIT
NO
d1d2 < S2
YES
Y = A{d1d2 - N1}
EXIT
NO
d1d2d3 < S3
YES
Y = A{d1d2d3 - N2}
EXIT
NO
d1d2d3d4 < S4
YES
Y = A{d1d2d3d4 - N3}
EXIT
NO
GENERATE
u = .d1d2...

CHAPTER 3

GENERAL TECHNIQUES FOR THE GENERATION OF PSEUDO-RANDOM NUMBERS HAVING A CONTINUOUS DISTRIBUTION FUNCTION

Certain techniques of a rather general nature are used in some of the routines described in this paper. They are especially well suited for use in very fast computer programs. The principle of each technique is described below and reference will be made to these techniques as they are used in the specific generator programs.

3.1 The Composition Technique

The fundamental procedure used for generating pseudo-random numbers from a continuous probability distribution, as suggested by Marsaglia [10], is based upon a decomposition of the density function f into a mixture of 3 densities:

$$f(t) = p_1 g_1(t) + p_2 g_2(t) + p_3 g_3(t) \tag{3.1.1}$$

where $p_1 > p_2 > p_3$, $p_1 + p_2 + p_3 = 1$ and p_1 is very close to 1.

A random number from f(t) is obtained as either a number from $g_1(t)$, from $g_2(t)$ or from $g_3(t)$ with respective probabilities p_1, p_2, p_3. Consider a density f(t) defined from all $t \geq a$ (figure 3.1).

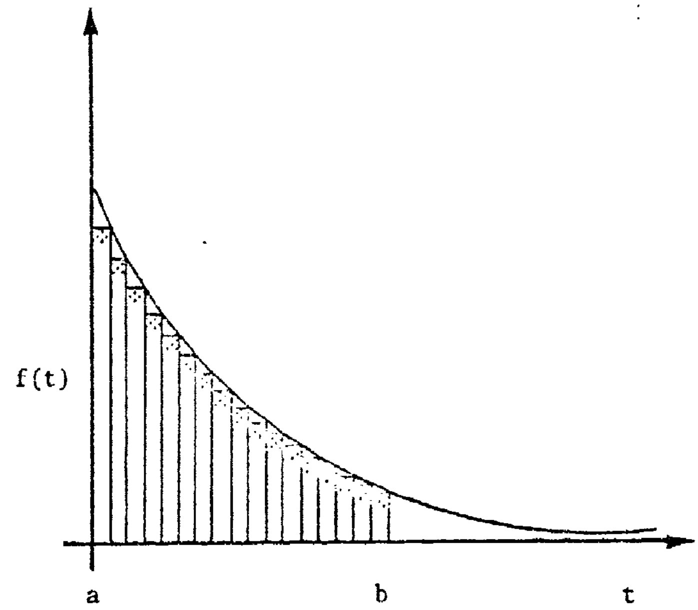


FIGURE 3.1

The density $g_1(t)$ is represented by the rectangles, appropriately standardized, including the shaded upper portions, and is defined for $a \leq t < b$. The width of the rectangles is Δ, a quantity whose value is dictated by the number system used within the computer. For a binary machine using hexadecimal arithmetic, $\Delta = 0.1_{(16)} = 0.0625_{(10)}$. The total area represented by the rectangles is p_1 which clearly is very close to 1. The density $g_2(t)$ is represented by the "triangular" regions, appropriately standardized, lying below $f(t)$ and above $g_1(t)$, and is also defined for $a \leq t < b$. The total area occupied by the "triangles" is p_2. The density $g_3(t)$ is defined for $t \geq b$ and represents

the tail of $f(t)$. Its area is p_3.

Random variables having density $g_1(t)$ may be rapidly generated as follows:

Define

$$N = (b-a)/\Delta \tag{3.1.2}$$

and

$$t_i = a + (i-1) \cdot \Delta \quad \text{for } i = 1,2,3, \ldots, N. \tag{3.1.3}$$

Now assign each discrete t_i a probability

$$P_i = \Delta \cdot f(t_{i+1}) = .\delta_{1i}\delta_{2i}\ldots . \tag{3.1.4}$$

The P_i's are simply the areas of the individual rectangles comprising $g_1(t)$. Based on (say) the high order 4 digits of the P_i's store the discrete t_i's in a table according to the technique described in Chapter 2 of this paper. Thus to generate a random variable Y from this portion of g_1 (unshaded in figure 3.1), generate a uniform (0,1) random number $u = .d_1d_2 \ldots .$ If

$$u < \sum_{i=1}^{N} .\delta_{1i}\delta_{2i}\delta_{3i}\delta_{4i}$$

then allow the 4 high order digits of u, $d_1d_2d_3d_4$, to locate a particular t_i from the discrete table, as described in Chapter 2, and set $Y = t_i + \Delta \cdot (.d_5d_6\ldots)$.

Random variables from the residual of g_1, corresponding to the remaining digits of the P_i's $(.0000\delta_{5i}\delta_{6i}\ldots)$ and represented by the shaded region in figure 3.1, are generated according to the following scheme: Let N, t_i, and P_i be defined as before (3.1.2), (3.1.3), and (3.1.4); and define:

$$H_i = \delta_{1i}\delta_{2i}\delta_{3i}\delta_{4i} \tag{3.1.5}$$

(area of the unshaded portion of the i^{th} rectangle),

$$R_i = P_i - H_i \tag{3.1.6}$$

(area of the shaded portion of the i^{th} rectangle),

$$T_i = F(t_{i+1}) - F(t_i) - P_i \tag{3.1.7}$$

where

$$F(t_i) = \int_a^{t_i} f(t_i)dt.$$

(T_i is the area of the "triangular" region above the i^{th} rectangle), and

$$P_i' = \sum_{i=1}^{N} H_i \tag{3.1.8}$$

(total area occupied by the unshaded rectangles). It is obvious from the above definitions that

$$p_2 = \sum_{i=1}^{N} T_i \quad , \tag{3.1.9}$$

$$p_1 = \sum_{i=1}^{N} P_i \quad , \text{ and} \tag{3.1.10}$$

$$p_1 - p_1' = \sum_{i=1}^{N} R_i \tag{3.1.11}$$

(See(3.1.1) for the significance of p_1 and p_2). Now store the t_i's in N consecutive locations. The order in which the t_i's are stored is not important*. However, for convenience let D(1) denote the t-value occupying

---

*They may be overlapped with the previously stored t-values in order to save storage space.

the first location, D(2) denote the t-value occupying the second location, etc. For each t_i define a pair of values C(k) and B(k) as follows:

$$C(1) = p_1' + R[D(1)], \tag{3.1.12}$$

$$B(k) = C(k) + T[D(k)], \text{ and} \tag{3.1.13}$$

$$C(k+1) = C(k) + R[D(k+1)] \tag{3.1.14}$$

for k = 1,2, ..., N. R[D(k)] and T[D(k)] denote respectively the area of the rectangle residue (shaded in figure 3.1) and the area of the "triangular" region corresponding to the t-value occupying location D(k). It is apparent that

$$B(N) = p_1 + p_2. \tag{3.1.15}$$

If u is a uniform (0,1) random number such that $p_1' \leq u < p_1 + p_2$, a random variable Y is generated from either the rectangular residues of g_1 or the "triangular" regions of g_2 as follows:

If $B(k-1) \leq u < C(k)$ then generate a new uniform (0,1) random number v and set $Y = D(k) + \Delta \cdot v$. A random variable generated in this fashion will have the density represented by the shaded rectangles in figure 3.1. If $C(k) \leq u < B(k)$, then the random variable Y must be generated from the "triangular" density of g_2 corresponding to the t-value occupying location D(k). The method used depends upon the particular function f(t). The same applies for random variables from g_3 for $t > b$.

The techniques used for the exponential and normal densities will be described subsequently.

Since p_1 is close to 1, most of the time a random variable from g_1 is generated. Though g_2 and g_3 may be complicated and require longer running time, the average time per generation is small since they must be handled so rarely.

3.2 The Acceptance-Rejection Method

Consider the density $h(t)$ defined on the interval (ξ_0,ξ_1). See figure 3.2. Now if u,v is a pair

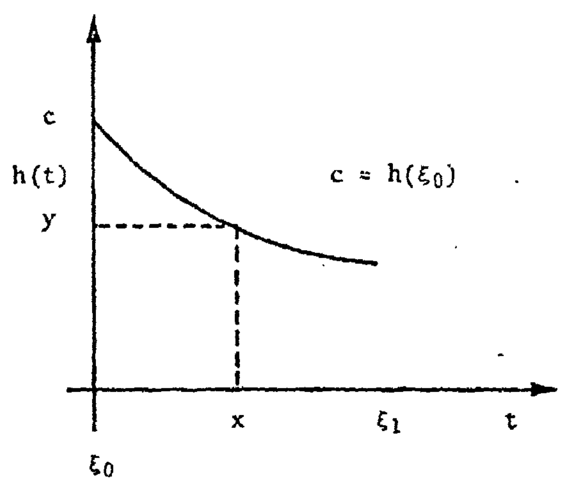


FIGURE 3.2

of independent uniform (0,1) random numbers, $x = \xi_0 + u \cdot (\xi_1-\xi_0)$ and $y = v \cdot c$ will have uniform densities on (ξ_0,ξ_1) and $(0,c)$ respectively. Suppose that $y \leq h(x)$. Then the conditional distribution function of x is given by

$$P[X \leq x \mid Y \leq h(x)] \propto \int_{\xi_0}^{x} \int_{0}^{h(t)} dydt = \int_{\xi_0}^{x} h(t)dt. \qquad (3.2.1)$$

Consequently the conditional density of x given $y \leq h(x)$ is $h(x)$.

When a pair of independent uniform (0,1) numbers, u and v, are generated and the resulting x and y satisfy the inequality $y \leq h(x)$, x is then a required random variate from $h(x)$. Otherwise the pair is rejected and a new pair is generated and checked. It is clear that the inequality will be satisfied with a high probability only if the value $c \cdot (\xi_1 - \xi_2)$ is close to one. If not, the procedure will produce a large proportion of inadmissable (u,v) pairs and reduce the efficiency of the scheme. Even when this proportion is small, the time required to check $y \leq h(x)$ is generally rather long compared with that of the table look-up procedures. Consequently, the acceptance-rejection method is best used for generating those infrequent values from the "triangular" and tail regions described in section 3.1.

3.3 Generation of Numbers with a Triangular Density

Let u and v be independent uniform (0,1) random numbers. Then $T = \min(u,v)$ has the distribution function

$$G(t) = 1 - P[T>t] = 1 - P[u>t,\ v>t] = 1 - (1-t)^2, \quad 0<t<1, \tag{3.3.1}$$

and the density function

$$g(t) = 2(1-t), \quad 0<t<1. \tag{3.3.2}$$

See figure 3.3a. A linear transformation $T' = a + bT$ produces a random variable with density

$$g(t') = \frac{2}{b} - \frac{2}{b^2}(t' - a), \quad a<t<a+b \tag{3.3.3}$$

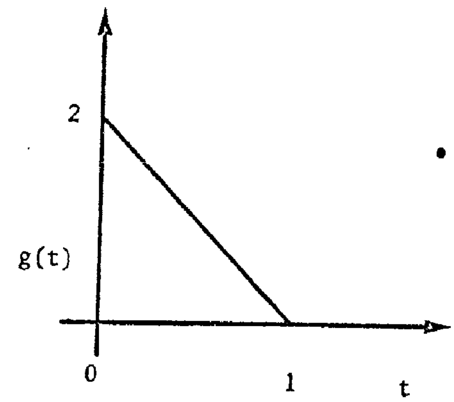


FIGURE 3.3(a)

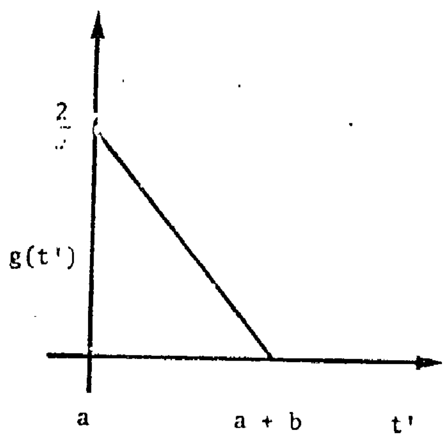


FIGURE 3.3(b)

3.4 The Distribution of the Minimum of a Random Number of Uniform (0,1) Random Variates: Some Special Results for the Exponential Distribution

Let $x = c \cdot \min(u_1, u_2, \ldots, u_n)$ where the u_i are random independent uniform (0,1) variates, c is a positive constant, and N is a discrete valued random variable with probability function $P(N=n) = q(n)$ for $n = 1,2, \ldots$. Then the distribution function of x can be expressed as follows:

$$F(x) = \sum_{n=1}^{\infty} P[X \leq x \mid N = n] \cdot P[N = n]$$

$$= \sum_{n=1}^{\infty} [1 - (1 - x/c)^n] \cdot q(n)$$

$$F(x) = 1 - \sum_{n=1}^{\infty} (1 - x/c)^n \cdot q(n) \qquad 0 < x < c \qquad (3.4.1)$$

Now consider the special case

$$q(n) = c^n/[n!(e^c-1)] \qquad n = 1,2, \ldots .$$

The distribution of x becomes

$$F(x) = 1 - \sum_{n=1}^{\infty} (c-x)^n/n!(e^c-1) = 1 - (e^{c-x}-1)/(e^c-1)$$

$$F(x) = (1-e^{-x})/(1-e^{-c}) \qquad 0 < x < c \qquad (3.4.3)$$

Thus x has the exponential distribution truncated on the right at the value c.

If, instead of the probability function given in (3.4.2), N has the distribution specified by

$$g(n) = \Delta^n/n!(e^{\Delta}-1-\Delta) \quad n = 2,3, \ldots, \tag{3.4.4}$$

where Δ is a constant as defined in section 3.1, then the distribution function of x becomes

$$F(x) = 1 - (e^{\Delta-x}-1-\Delta+x)/(e^{\Delta}-1-\Delta), \quad 0<x<\Delta, \tag{3.4.5}$$

with density

$$f(x) = (e^{-x}-e^{-\Delta})/[1 - e^{-\Delta}(1+\Delta)] \ , \quad 0<x<\Delta. \tag{3.4.6}$$

This is the distribution function of a random variable from the "tooth" of the exponential distribution between 0 and Δ. See figures 3.4(a) and 3.4(b).

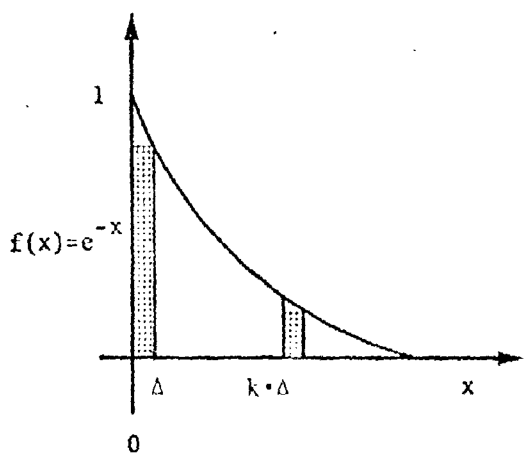


FIGURE 3.4(a)

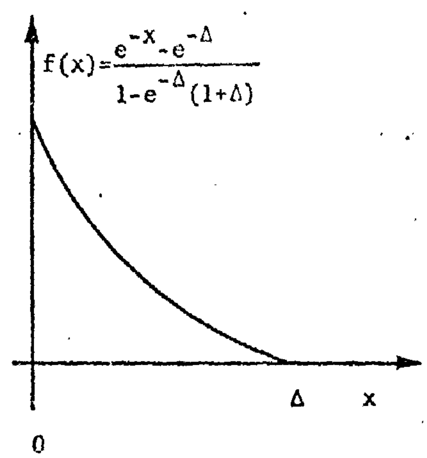


FIGURE 3.4(b)

Figure 3.4(c) shows the density of a random variable from any other such "tooth" defined by $k \cdot \Delta < z < (k+1) \cdot \Delta$.

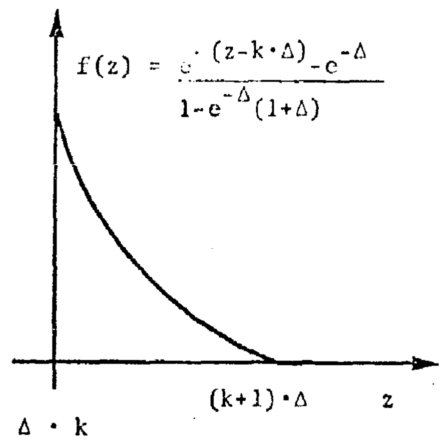


FIGURE 3.4(c)

It is obvious that all such densities are identical, a unique characteristic of the exponential distribution. Consequently, a random variable from the $(k+1)^{st}$ "tooth" is always produced by taking $z = k \cdot \Delta + x$ where the value $k \cdot \Delta$ is chosen by the technique described in section 3.1 and x is generated by taking Δ times the minimum of N uniform (0,1) random numbers where N has the distribution (3.4.4). Its expected value when $\Delta = 0.1_{(16)}$ is 2.02 or in general

$$E[N] = \Delta \cdot (e^{\Delta}-1)/(e^{\Delta}-1-\Delta) \qquad (3.4.7)$$

The same unique "no memory" characteristic of the exponential distribution used above is also used for obtaining random variates from g_3, the tail of the exponential density where $t > c$.

Let $Y = W + X$ where W is discrete valued with probability function

$$P[W = w = c \cdot k] = Ae^{-w} = Ae^{-c \cdot k}, \quad k = 1,2,3, \ldots, \qquad (3.4.8)$$

$A = (1 - e^{-c})/e^{-c}$ and x is independent of w and has the distribution function (3.4.3). The distribution function of Y can be written as

$$G(y) = P[Y \leq y = w + x] = P[Y \leq w - c] + P[W = w, X \leq x] \qquad (3.4.9)$$

Since the distribution function of W is

$$P[W \leq w = c \cdot k] = A \sum_{j=1}^{k} e^{-j \cdot c} = 1 - e^{-c \cdot k}$$

the expression for $G(y)$ becomes

$$G(y) = 1 - e^{-(w-c)} + Ae^{-w}(1-e^{-x})/(1-e^{-c}) = 1 - e^{c-y}, \qquad (3.4.11)$$

$$y > c.$$

This is the required distribution function for variates from the density e^{-y} truncated on the left at $y = c$. Thus values of t from g_3 in the exponential case are generated as the sum of a discrete variate taking values c, $2c$, $3c$, ... and a continuous variate from the truncated exponential density on $(0,c)$.

3.5 Generating Numbers from a 'Nearly Triangular' Density: Application to the Half Normal Density

The results of this section are applied to the half normal density

$$f(t) = \sqrt{2/\pi}\; e^{-\frac{1}{2}t^2} \qquad 0 < t < \infty \qquad (3.5.1)$$

for the generation of random normal deviates to be described in chapter 4. Consider the concave and convex triangular densities (members of g_2) depicted in figures 3.5(a) and 3.5(b).

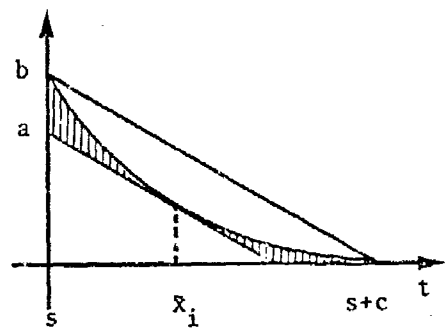


FIGURE 3.5(a)

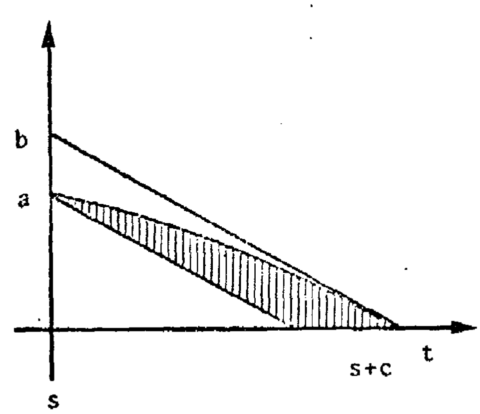


FIGURE 3.5(b)

The parallel chords and tangents enclosing f(t) are determined by construction as indicated in the figures. In both cases, the inner right triangle represents most of the area under the density F(t). This will be true in general as long as the ratio a/b is close to one. A random variate from the density represented by this triangle is generated as described in section 3.3. Infrequently, a random variate from the shaded area must be generated by the acceptance-rejection technique described in section 3.2. Marsaglia et al. [12] have combined these two procedures as follows.

1) Generate independent uniform (0,1) random variables u and v.

2) If $\max(u,v) \leq a/b$, put $t = s + c \cdot \min(u,v)$

3) If not, test $b|u-v| \leq f(s + c \cdot \min(u,v))$.

   If yes, put $t = s + c \cdot \min(u,v)$.

   If no, go to step 1 and try again.

In order to show that this procedure produces a variate t with the required density, we conveniently take $s = 0$ without loss of generality and proceed as follows.

Define

$$\begin{aligned} m &= \min(u,v) \\ M &= \max(u,v) \\ x &= c \cdot m \\ y &= b(M-m) = b(u-v) \end{aligned} \qquad (3.5.2)$$

Then the pair (x,y) is uniformly distributed over the triangle in figure 3.5(c)

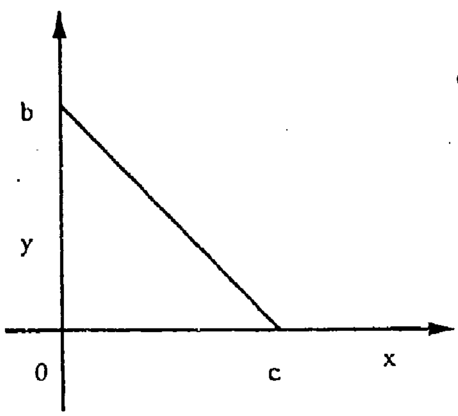


FIGURE 3.5(c)

To verify this, consider the joint distribution function of X and Y:

$$F(x,y) = P[X \le x,\ Y \le y] = P[m \le x/c,\ M - m \le y/b]$$

This is just the area of the cross hatched region in figure 3.5(d) which is seen to be

$$F(x,y) = 2 \cdot (y/b) \cdot (x/c) = 2xy/bc\ ,\quad 0<x<c\ ,\quad 0<y<b\ . \tag{3.5.3}$$

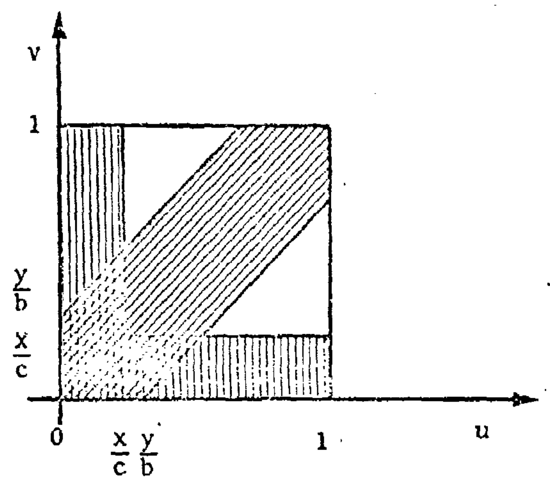


FIGURE 3.5(d)

Thus, (x,y) is a point randomly chosen from the larger triangle in figures 3.5(a) and 3.5(b). By the acceptance-rejection technique, if this point falls within the smaller triangle of figures 3.5(a) and 3.5(b), the value $x = t$ has the triangular density $-bx/c + a$. Now the condition $\max(u,v) \le a/b$ is equivalent to:

$$bM \le a$$

$$b(M-m) \le a - bm = -bx/c + a$$

or

$$y \leq -bx/c + a \tag{3.5.4}$$

which means that (x,y) falls in the small triangle. If this condition is not satisfied, a second check is made to determine whether the point (x,y) falls within the shaded area between $f(t)$ and y_2 in figures 3.5(a) and 3.5(b). If so, then clearly

$$y = b(M - m) = b|u - v| \leq f(x) = f(c \cdot \min(u,v)) \tag{3.5.5}$$

as required by the second test. The acceptance-rejection principle insures that every x value passing at least one of these two tests will have the density $f(t)$. The first test is rapidly made; the second test involves evaluation of $f(x)$ and is time consuming. The first test will be passed with a probability

$$P[M \leq a/b] = (a/b)^2$$

as given by the distribution function of $M = \max(u,v)$. It is therefore desirable to have $(a/b)^2$ close to 1 to achieve a short average execution time. This will be the case if $f(t)$ is "nearly triangular."

For the half normal density the a_i/b_i ratios are obtained as follows.

Case (a) "Concave Triangles" (See figure 3.5(a)).

Let $t_i = i \cdot c = s$

$$t_{i+1} = (i+1)c = s + c, \quad i = 0,1,2, \ldots \tag{3.5.6}$$

The equation of the chord and the tangent are respectively:

$$y_1 = - b_i(\quad s)/c + b_i$$

$$y_2 = - b_i(x-s)/c + a_i \qquad (3.5.7)$$

The value of b_i is clearly

$$b_i = f(t_i) - f(t_{i+1}) = f(s) - f(s+c) \qquad (3.5.8)$$

The slope of both lines, $- b_i/c$, must equal $f'(\bar{x}_i)$. Thus

$$-b_i/c = - \sqrt{2/\pi}\, x_i e^{-\frac{1}{2}x_i^2} = - \bar{x}_i f(\bar{x}_i) \qquad (3.5.9)$$

This equation must be solved iteratively for $\bar{x}_i$, the abscissa of the point of tangency. Finally, this value is substituted into the tangent's equation to give

$$f(\bar{x}_i) = f(t_{i+1}) = (- b_i/c)(\bar{x}_i - s) + a_i. \qquad (3.5.10)$$

Substituting the expression for b_i/c and solving for a_i gives

$$a_i = f(\bar{x}_i)[1 + \bar{x}_i(\bar{x}_i - s)] - f(s+c). \qquad (3.5.11)$$

Consequently for "concave triangles"

$$a_i/b_i = [f(\bar{x}_i)\{1 + \bar{x}_i(\bar{x}_i - s)\} - f(s+c)]/[f(s) - f(s+c)] \qquad (3.5.12)$$

Case (b) "Convex Triangles" (See figure 3.5(b))

In this case the point of tangency occurs at the end of the interval where $x_i = t_{i+1} = s + c$. The slope of both lines is given by

$$- b_i/c = f'(s+c)$$

$$b = -cf'(s+c). \tag{3.5.13}$$

or

The value of a_i is simply

$$f(t_i) - f(t_{i+1}) = f(s) - f(s+c).$$

Consequently, for "convex triangles"

$$a_i/b_i = [f(s) - f(s+c)]/[-cf'(s+c)] \tag{3.5.14}$$

3.6 Generation of Random Variates from the Tail of the Half Normal Distribution

The procedure suggested by Marsaglia et al. [12] for the generation of random variates from the tail of the half normal distribution is based on the rejection principle described in section 3.2. The procedure is to generate a pair of independent half normal variates x_1 and x_2 such that the point (x_1, x_2) lies outside the quarter circle (see figure 3.6(a)).

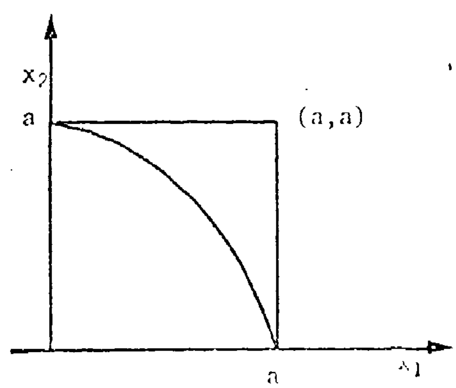


FIGURE 3.6(a)

This is done by generating pairs of uniform (0,1) random numbers u_1 and u_2 until $0 < u_1^2 + u_2^2 \leq 1$ and setting

$$x_1 = \sqrt{a^2 - 2\ln(u_1^2 + u_2^2)} \cdot \sqrt{u_1^2/(u_1^2 + u_2^2)} \qquad (3.6.1)$$

$$x_2 = \sqrt{a^2 - 2\ln(u_1^2 + u_2^2)} \cdot \sqrt{u_2^2/(u_1^2 + u_2^2)} \qquad (3.6.2)$$

To show that the point (x_1,x_2) lies outside the quarter-circle, let $w = -\ln(u_1^2 + u_2^2)$ where $0 < u_1^2 + u_2^2 \leq 1$. Then take the sum $x_1^2 + x_2^2 = a^2 + 2w$ where $0 \leq w < \infty$ so that $x_1^2 + x_2^2 \geq a^2$. If the point (x_1,x_2), generated in this fashion, lies within the square (figure 3.6(a)), it is rejected and a new pair of uniform (0,1) random numbers is generated and tested. Otherwise the variable, x_1 or x_2, whose value exceeds a is taken as the required random number.

In order to show that this procedure produces the desired result, let u and u be a pair of uniform (0,1) random numbers conditioned by $0 < u_1^2 + u_2^2 \leq 1$. Then the joint density of u_1 and u_2 is

$$f(u_1,u_2) = 4/\pi, \quad 0 < u_1 < 1, \quad 0 < u_2 < 1, \quad 0 < u_1^2 + u_2^2 \leq 1. \qquad (3.6.3)$$

Now define:

$$X = u_1^2 + u_2^2, \quad 0 < X < 1, \text{ and} \qquad (3.6.4)$$

$$Y = u_2/u_1, \quad 0 < Y < \infty. \qquad (3.6.5)$$

Then the probability distribution of X is

$$F(x) = P(X \leq x) = (\pi \cdot x/4)/(\pi \cdot 1^2/4) = x \qquad (3.6.6)$$

and the density of x is

$$f(x) = 1, \quad 0 < x < 1. \tag{3.6.7}$$

Consequently X is uniform on (01). The joint density of X and Y is:

$$g(x,y) = f(u,v) \cdot |J| = (4/\pi)[1/2(1+y^2)] = 2/\pi(1+y^2) = g_1(x) \cdot g_2(y). \tag{3.6.8}$$

Hence X and Y are stochastically independent. Substitution of (3.6.4) and (3.6.5) into (3.6.1) and (3.6.2) yields:

$$x_1 = \sqrt{a^2 - 2\ln X} \cdot \sqrt{1/(1+Y^2)} \tag{3.6.9}$$

$$x_2 = \sqrt{a^2 - 2\ln X} \cdot \sqrt{Y^2/(1+Y^2)}. \tag{3.6.10}$$

The inverse transformation is:

$$X = e^{-\frac{1}{2}(X_1^2+X_2^2-a^2)} \tag{3.6.11}$$

$$Y = X_2/X_1 \tag{3.6.12}$$

The joint density of X_1 and X_2 is given by:

$$\begin{aligned} f(x_1,x_2) &= g(x,y) \cdot |J| \\ &= [2/\pi(1+y^2)](1+y^2) \cdot e^{-\frac{1}{2}(x_1^2+x_2^2-a^2)} \\ &= (\sqrt{2/\pi}\, e^{-\frac{1}{2}x_1^2})(\sqrt{2/\pi}\, e^{-\frac{1}{2}x_2^2})\, e^{a^2/2} \end{aligned} \tag{3.6.13}$$

$$a^2 \leq x_1^2 + x_2^2 < \infty .$$

Clearly x_1 and x_2 are independent random variables from the tail of the half normal distribution ($x_1, x_2 > a$).

CHAPTER 4

GENERATION OF PSEUDO-RANDOM NUMBERS FROM THE EXPONENTIAL, GAMMA, AND NORMAL DISTRIBUTIONS

Before presenting the procedures for the generation of random numbers from the exponential, gamma, and normal distributions, a simple example is given demonstrating the use of the techniques discussed in Chapter 3 for obtaining random variates from any continuous distribution defined on a finite interval. In the interest of clarity all numbers are expressed to the base 10. Consider the distribution defined by

$$F(y) = (y/2)(3-y^2) \quad 0 < y < 1 \tag{4.0.1}$$

with density

$$f(y) = (3/2)(1-y^2) \quad 0 < y < 1. \tag{4.0.2}$$

Using the composition technique presented in 3.1, $f(y)$ is represented as a mixture of 2 densities (since there is no tail):

$$f(y) = p_1 g_1(y) + p_2 g_2(y). \tag{4.0.3}$$

As before, g_1 is a series of rectangles and g_2 represents the nearly triangular regions between g_1 and f. See figures 4.0(a), (b), and (c).

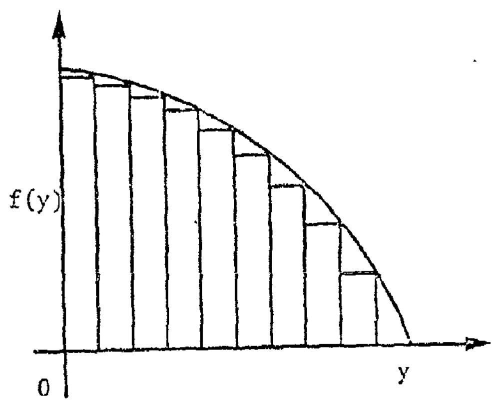


FIGURE 4.0(a)

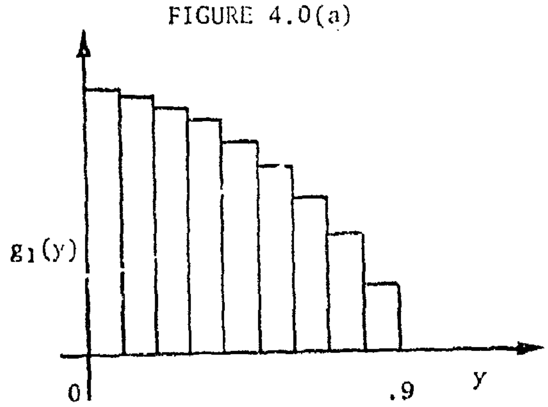


FIGURE 4.0(b)

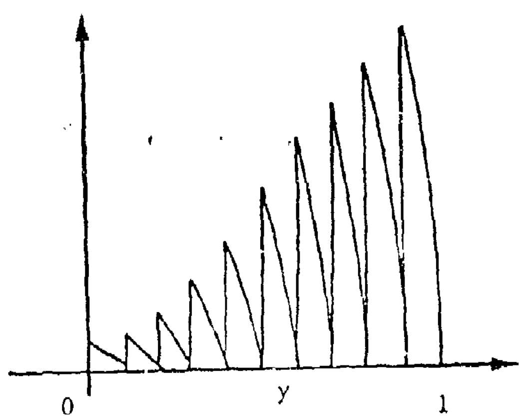


FIGURE 4.0(c)

As described in 3.1, define discrete variables

x_i = (i-1) · Δ for i=1,2,...,10 and in this example $\Delta=0.1_{(10)}$. (4.0.4)

Now assign each x_i a probability P_i as follows:

$$P_i = \Delta \cdot f(x_{i+1}). \tag{4.0.5}$$

Now define a value T_i as given in (3.1.7), the area of the triangular region above the i^{th} rectangle:

$$T_i = F(x_{i+1}) - F(x_i) - P_i. \tag{4.0.6}$$

These values are recorded in table 4.0.1. At this point it is obvious that the values of p_1 and p_2, defined in (4.0.3), are p_1 = 0.9225 and p_2 = 0.0775. Next, the four sets of the discrete values are stored, based on the high order of digits of the P_i's.

x_i	$P_i = 0.1 \cdot f(x_{i+1})$	$F(x_{i+1}) - F(x_i)$	$T_i = F(x_{i+1}) - F(x_i) - P_i$
0.0	0.1485	0.1495	0.0010
0.1	0.1440	0.1465	0.0025
0.2	0.1365	0.1405	0.0040
0.3	0.1260	0.1315	0.0055
0.4	0.1125	0.1195	0.0070
0.5	0.0960	0.1045	0.0085
0.6	0.0765	0.0865	0.0100
0.7	0.0540	0.0655	0.0115
0.8	0.0285	0.0415	0.0130
0.9	0.0000	0.0145	0.0145
Total	0.9225	1.0	0.0775

TABLE 4.0.1

Since the P_i's have only 4 digits, application of this technique will account for all of g_1 and there is no rectangular residue in this simple example. Since the composition of these 4 sets is constant (the distribution is parameter free), the sets may be overlapped and considerable storage space saved. Looking at table 4.0.2, set 1 occupies locations 2 through 6, set 2 occupies locations 4 through 40, set 3 occupies locations 18 through 67, and set 4 occupies locations 46 through 70. Locations 0 through 9 are also used for the generation of variates from g_2 , as described subsequently.

TABLE 4.0.2

A-TABLE

Loc	Con	Loc	Con	Loc	Con	Loc	Con	Loc	Con	Loc	Con	Loc	Con
0	0.9	10	0.6	20	0.7	30	0.5	40	0	50	0.8	60	0.2
1	0.8	11	0.6	21	0.7	31	0.3	41	0.8	51	0.6	61	0.2
2	0.3	12	0.6	22	0.7	32	0.3	42	0.3	52	0.6	62	0.2
3	0.1	13	0.6	23	0.7	33	0.2	43	0.3	53	0.6	63	0
4	0.4	14	0.6	24	0.6	34	0.1	44	0.3	54	0.6	64	0
5	0.2	15	0.5	25	0.5	35	0.1	45	0.3	55	0.6	65	0
6	0	16	0.5	26	0.5	36	0.1	46	0.8	56	0.4	66	0
7	0.7	17	0.2	27	0.5	37	0.1	47	0.8	57	0.4	67	0
8	0.6	18	0.8	28	0.5	38	0	48	0.8	58	0.2	68	0.4
9	0.5	19	0.8	29	0.5	39	0	49	0.8	59	0.2	69	0.4
												70	0.4

As presented in Chapter 2, define:

$$Q_0 = 0, \quad Q_r = 10^{-r} \cdot \sum_{i=1}^{10} \delta_{ri} \tag{4.0.7}$$

and

$$\Pi_0 = 0, \quad \Pi_r = \sum_{j=1}^{r} \sum_{i=1}^{10} \delta_{ji} \quad \text{for } r=1,2,3,4. \tag{4.0.8}$$

Using the above definitions define:

$$S_0 = 0, \quad S_k = 10^k \sum_{r=1}^{k} Q_r \tag{4.0.9}$$

and

$$N_k = \Pi_{k-1} - 10 \cdot S_{k-1} \quad \text{for } k=1,2,3,4. \tag{4.0.10}$$

The N_k's are identical to the quantity

$$\Pi_{s-1} + \beta^s \cdot \sum_{r=0}^{s} P_r$$

in (2.0.3). Because of overlapping between the four sets, the N_k's must be adjusted. Their values and the values of the S_k's are recorded in table 4.0.3.

S-TABLE		N-TABLE	
Loc	Con	Loc	Con
0	5	0	2
1	87	1	-46
2	920	2	-852
3	9225	3	-9154

Table 4.0.3

The procedure for generating random numbers from g_1 is:

1) Generate a uniform (0,1) random number $u = 0.d_1d_2\ldots$.

2) If $d_1 < S(1)$, set $Y = A\{d_1 + N(1)\} + 0.1 \cdot (.d_5d_6\ldots)$; otherwise go to (3).

3) If $d_1d_2 < S(2)$, set $Y = A\{d_1d_2 + N(2)\} + 0.1 \cdot (.d_5d_6\ldots)$; otherwise go to (4).

4) If $d_1d_2d_3 < S(3)$, set $Y = A\{d_1d_2d_3 + N(3)\} + 0.1 \cdot (.d_5d_6\ldots)$; otherwise go to (5).

5) If $d_1d_2d_3d_4 < S(4)$, set $Y = A\{d_1d_2d_3d_4 + N(4)\} + 0.1 \cdot (.d_5d_6\ldots)$; otherwise generate a random variate from g_2.

In order to generate random variates from g_2, define:

$$C(1) = S_4 + T[A(1)] \text{ and} \tag{4.0.11}$$

$$C(k+1) = C(k) + T[A(k)] \quad \text{for } k=1,2,\ldots 9. \tag{4.0.12}$$

This is identical to the definition given in (3.1.13), except that in this example there is no rectangular residue. The notation T[A(k)] denotes the area of the triangular region (the T_i's in table 4.0.1) corresponding to the discrete value stored in the k^{th} location of the A-table (Table 4.0.2). The values of the C(k)'s are given in table 4.0.4. If u is a uniform (0,1) random number such that $u \geq S_4$, the proper triangle of g_2 may be chosen simply by testing:

$u < C(k)$ until the condition is satisfied (note that $C_{(10)} = 1.0$)

The discrete value occupying location k of the A-table denotes the correct triangle.

Having selected the proper triangle of g_2, a random variate having the density of that triangle is to be generated. This is accomplished

through the use of the acceptance-rejection principle discussed in 3.2 and the technique given in 3.5. Consider a particular triangle or tooth from g_2 (figure 4.0(d)). The tangent and the chord are constructed as indicated in 3.5

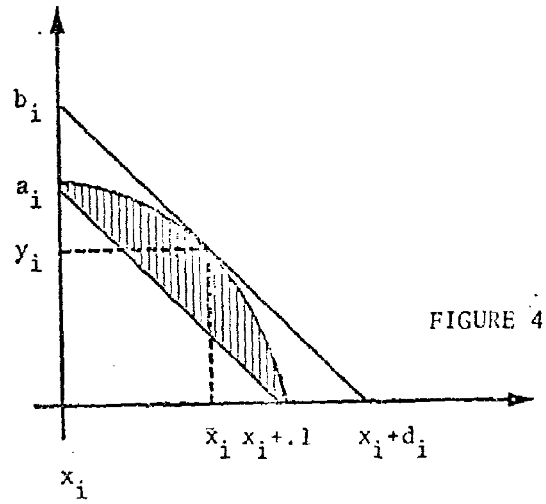


FIGURE 4.0(d)

The inner right triangle encloses most of the area under f(x). A random variate from the density of this triangle is generated as described in 3.3. Occasionally, a random variate from the shaded region must be generated by the acceptance-rejection technique. The values of a_i, b_i, and d_i are obtained as follows:

$$-(a_i/0.1) = f'(\bar{x}_i) = 3\bar{x}_i \tag{4.0.13}$$

where x_i is the abscissa of the point of tangency and

$$a_i = f(x_i) - f(x_{i+1}). \tag{4.0.14}$$

Substituting (4.0.14) into (4.0.13) and solving for $\tilde{x}_i$ yields:

$$\tilde{x}_i = [f(x_i) - f(x_{i+1})]/0.3 \qquad (4.0.15)$$

The ordinate of the point of tangency is:

$$y_i = f(\tilde{x}_i) - f(x_{i+1}). \qquad (4.0.16)$$

At $x = \tilde{x}_i$, $y_1 = y_i$ and

$$b_i = y_i + (a_i/0.1)(\tilde{x}_i - x_i) \qquad (4.0.17)$$

The value of d_i may be obtained by noting that:

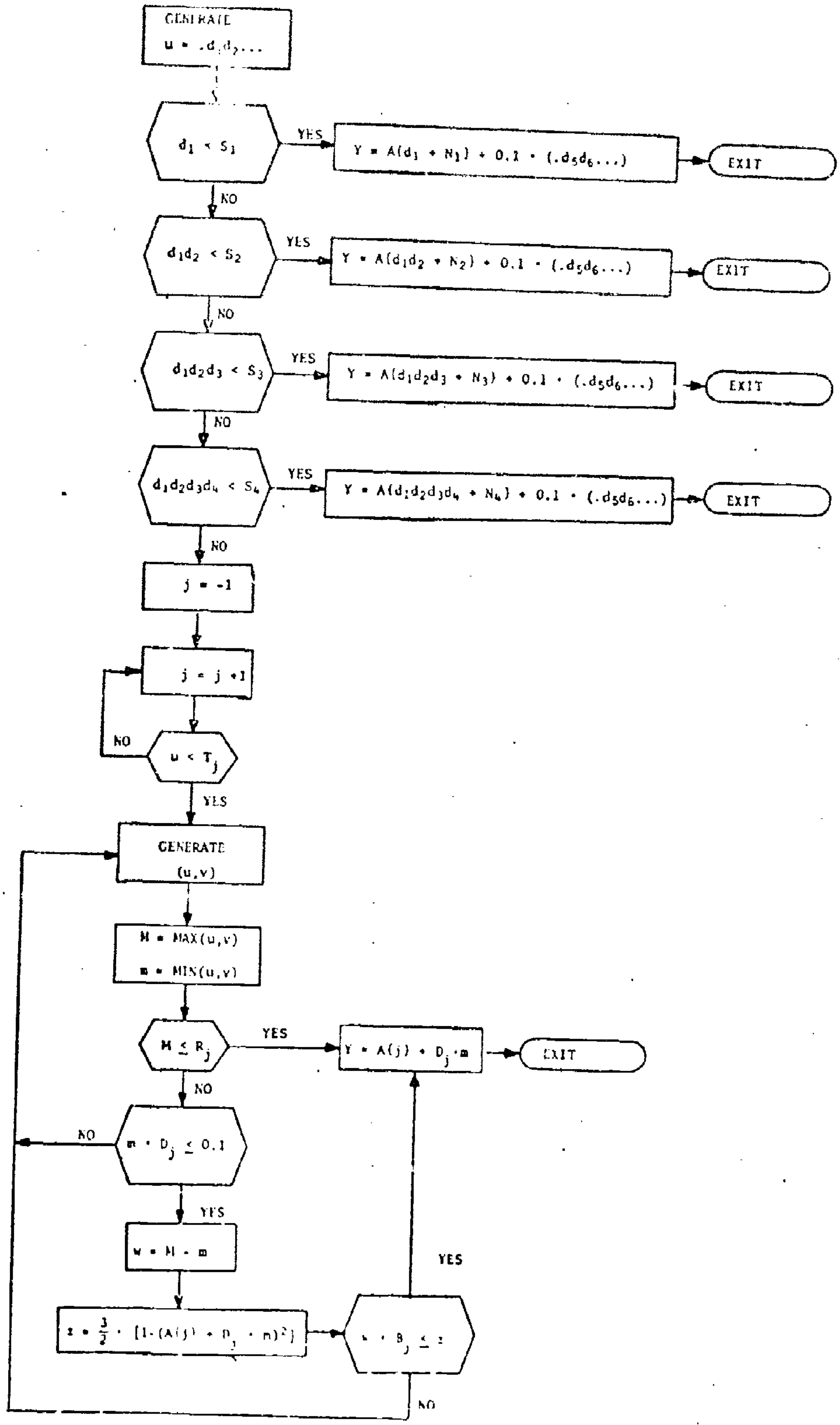
$$(a_i/0.1) = (b_i/d_i) \text{ and hence } d_i = (0.1 \cdot b_i/a_i). \qquad (4.0.18)$$

The values of a_i/b_i, d_i, and b_i are recorded in table 4.0.4 in the R-table, D-table, and B-table respectively.

C-Table		R-Table		D-Table		B-Table	
Loc	Con	Loc	Con	Loc	Con	Loc	Con
0	.9370	0	.9870	0	.1013	0	.28875
1	.9500	1	.9855	1	.1015	1	.25875
2	.9555	2	.9655	2	.1036	2	.10875
3	.9580	3	.9231	3	.1083	3	.04875
4	.9650	4	.9730	4	.1028	4	.13875
5	.9690	5	.9524	5	.1050	5	.07875
6	.9700	6	.8000	6	.1250	6	.01875
7	.9815	7	.9836	7	.1017	7	.22875
8	.9915	8	.9811	8	.1019	8	.19875
9	1.0000	9	.9778	9	.1023	9	.16875

Table 4.0.4

FIGURE 4.0(e)
Flowchart for the Generation of Random
Variates from
$f(y) = (3/2)(1 - y^2), \quad 0 < y < 1$


GENERATE
u = .d1d2...
d1 < S1
YES
Y = A(d1 + N1) + 0.1 · (.d5d6...)
EXIT
NO
d1d2 < S2
YES
Y = A(d1d2 + N2) + 0.1 · (.d5d6...)
EXIT
NO
d1d2d3 < S3
YES
Y = A(d1d2d3 + N3) + 0.1 · (.d5d6...)
EXIT
NO
d1d2d3d4 < S4
YES
Y = A(d1d2d3d4 + N4) + 0.1 · (.d5d6...)
EXIT
NO
j = -1
j = j +1
NO
u < Tj
YES
GENERATE
(u,v)
M = MAX(u,v)
m = MIN(u,v)
M ≤ Rj
YES
Y = A(j) + Dj ·m
EXIT
NO
NO
m · Dj ≤ 0.1
YES
w = M - m
YES
NO

The procedure for obtaining random variates from g_2 is:

1) Let u be a uniform (0,1) random number such that $u \geq S_4$.
2) Test $u < C(k)$ for $k = 0, 1, \ldots, 9$ until the inequality is satisfied.
3) Generate independent uniform (0,1) random numbers u_1 and u_2.
4) If $\max(u_1, u_2) \leq R(k)$, then set $Y = A(k) + D(k) \cdot \min(u_1, u_2)$; otherwise go to (5).
5) If $B(k) \cdot |u_1 - u_2| \leq f[A(k) + D(k) \cdot \min(u_1, u_2)]$, set $Y = A(k) + D(k) \cdot \min(u_1, u_2)$; otherwise go to step (3) and try again.

A schematic of the entire procedure for generating random variates from the density $f(y)$, given in (4.0.2), is presented in figure 4.0(e).

4.1 Generation of Exponentially Distributed Random Numbers

Applying the composition principle presented in 3.1, the exponential density function,

$$f(t) = e^{-t}, \quad 0 \leq t \leq \infty, \tag{4.1.1}$$

is represented as a mixture of three densities

$$f(t) = p_1 g_1(t) + p_2 g_2(t) + p_3 g_3(t). \tag{4.1.2}$$

As before, g_1 is a series of rectangles defined for $t \leq 4$, g_2 represents the toothlike regions between g_1 and f defined for $t \leq 4$, and g_3 is the tail of f for $t \geq 4$. The probabilities p_1, p_2, and p_3 are respec-

tively the areas represented by g_1, g_2, and g_3.

As described in 3.1 a random variable Y having density g_1 is generated as follows:

Define

$M = 4/\Delta$, where $\Delta = 10^{-1}$ for a decimal machine or $\Delta = 16^{-1}$ for a binary machine; and discrete variates (4.1.3)

$t_i = (i-1) \cdot \Delta$ for $i = 1,2, \ldots, M$. (4.1.4)

Each t_i is assigned a probability

$$P_i = \Delta \cdot f(t_{i+1}) = \Delta \cdot e^{-t_{i+1}} = \delta_{1i}\delta_{2i}\delta_{3i}\cdots \qquad (4.1.5)$$

Now, using the high order 4 octal digits (binary machine) or the high order 3 decimal digits (decimal machine), store the t_i's according to the technique described in Chapter 2. As in the case of the example given in the beginning of this chapter, the sets of discrete variates are stored in the manner that permits maximum overlap. See the D-table in the program listing of GEN1 in the appendix; set 1 occupies locations 49A thru 4BE, set 2 occupies locations 40C through 4AC, and set 3 occupies locations 37D through 453. Locations 374 through 3B5 are used for the generation of variates from g_2 and the residual of g_1 as described in 3.1.

At this point define:

$$Q_0 = 0, \quad Q_r = \beta^{-r} \cdot \sum_{i=1}^{M} \delta_{ri} \qquad (4.1.6)$$

$$\Pi_0 = 0, \quad \Pi = \sum_{j=1}^{r} \sum_{i=1}^{M} \delta_{ji} \quad \text{for } r = 1,2,3,4 \qquad (4.1.7)$$

for binary machines where the δ's are octal digits and $\beta = 8$ or $r = 1,2,3$

for decimal machines where the δ's are decimal digits and $\beta = 10$. Also define:

$$S_0 = 0, \; S_k = \beta^k \cdot \sum_{r=1}^{k} Q_r \tag{4.1.8}$$

and

$$N_k = \Pi_{k-1} - \beta \cdot S_{k-1} \text{ for } k = 1,2,3,4 \tag{4.1.9}$$

in the binary case ($\beta = 8$) or $k = 1,2,3$ in the decimal case ($\beta = 10$). The S-values and the N-values are recorded in the program listing of GEN1 in the appendix. Note that the N-values have been adjusted to allow for overlap between the sets of the D-table.

The procedure for the generation of random variates from this truncated portion of g_1 is: (binary case)

1) Generate a uniform (0,1) random number $u = .d_1d_2\ldots$.
2) If $d_1d_2 < S_2$, set $Y = D\{d_1d_2 + N_2\} + \Delta \cdot (.d_5d_6\ldots)$; otherwise go to (3).
3) If $d_1d_2d_3 < S_3$, set $Y = D\{d_1d_2d_3 + N_3\} + \Delta \cdot (.d_5d_6\ldots)$; otherwise go to (4).
4) If $d_1d_2d_3d_4 < S_4$, set $Y = D\{d_1d_2d_3d_4 + N_4\} + \Delta \cdot (.d_5d_6\ldots)$; otherwise generate a random variate from the residual of g_1, g_2, or g_3.

Note that in the above procedure the testing begins with S_2. This is because $\delta_{1i} = 0$ for all i since $0 < f(x) < 1$ for $0 < x < 4$.

Random variates from the residual of g_1 and from g_2 are generated by first selecting one of the rectangles of g_1 with appropriate probability and then testing to determine whether the required variate should

come from the tooth or from the residual corresponding to that rectangle. This is accomplished as follows:

In M consecutive locations store each of the t_i's once (locations 374 through 3B3 of the D-table in the listing of GEN1 in the appendix). For each t_i define a pair of values as given in (3.1.12), (3.1.13), and (3.1.14)

$$C(1) = S_4 + R[D(1)], \tag{4.1.10}$$

$$B(k) = C(k) + T[D(k)] \text{ and} \tag{4.1.11}$$

$$C(k+1) = C(k) + R[D(k+1)]. \tag{4.1.12}$$

In the exponential case

$$T[D(k)] = e^{-D(k)} - (1 + \Delta) \cdot e^{-(D(k)+\Delta)} \tag{4.1.13}$$

and

$$R[D(k)] = P[D(k)] - .\delta[1,D(k)]\delta[2,D(k)]\delta[3,D(k)]\delta[4,D(k)], \tag{4.1.14}$$

where $P[D(k)]$ is as defined in (4.1.5). The values $T[D(k)]$ and $R[D(k)]$ denote respectively the areas of the triangular region and rectangular residue lost upon truncation of $P[D(k)]$ corresponding to the t-value occupying location $D(k)$. The values of the $B(k)$'s and the $C(k)$'s are also recorded in the listing of GEN1 in the appendix. A random variate with a rectangular distribution is generated as described in 3.1, and in the exponential case a variate having the density of one of the teeth of g_2 is generated by the technique given in 3.4. The procedure for generating random variates from either the residual of g_1 or from g_2 is summarized as follows:

1) Let u be a uniform (0,1) random number such that

$S_4 \leq u < p_1 + p_2 = 1 - e^{-4}$.

2) Test $u < B(J)$ for $J = 1,2, \ldots, \overline{M}$, until the inequality is satisfied (note $B(M) = 1 - e^{-4}$) for same $J = J'$.

3) If $u < C(J')$, generate a new uniform (0,1) random number v and set $Y = D(J') + \Delta \cdot v$ (this produces a number from g_1); otherwise go to (4).

4) Generate a new uniform (0,1) random number v and test

$v < Q(n) = \sum_{j=2}^{n} \Delta^j/[j!(e^{\Delta} - 1 - \Delta)]$ for $n = 2,3, \ldots$

until the inequality is satisfied for $n = n'$.

5) Generate n' independent uniform (0,1) random numbers and set $Y = D(J') + \Delta \cdot \min(u_1, u_2, \ldots, u_{n'})$. This produces a number from g_2.

A random variate Y having density g_3 is generated as a sum

$$Y = W + X \tag{4.1.15}$$

where X is exponential on (0,4) and W has the distribution

$$P[W = w = 4 \cdot k] = e^{-4k}(e^4 - 1), \quad k = 1,2, \ldots. \tag{4.1.16}$$

The procedure for generating a random variate from g_3 is summarized as follows:

1) Let u be a uniform (0,1) random number such that $u \geq p_1 + p_2 = 1 - e^{-4}$.

2) Test $u < 1 - e^{-4(k+1)}$ for $k = 1,2, \ldots$ until the inequality is satisfied for some $k = k'$.

3) Set $W = 4 \cdot k'$ and generate a new uniform (0,1) random number v.

4) Set $u = (p_1 + p_2) \cdot v = (1 - e^{-4}) \cdot v$ and enter the procedure for generating exponentially distributed random variates on the interval (0,4) with $u = (1 - e^{-4}) \cdot v$ as the initial uniform number and generate X.

5) Set $Y = W + X$.

In [13] Marsaglia presents a schematic and the necessary constants for the generation of random variates from the exponential distribution. Application of this method results in a very fast generator program. As is evident from the discussion herein the method is also exact; the accuracy of the result is dependent only on the word size of the computer.

4.2 Generation of Gamma Distributed Random Variates

In order to produce random variates having the gamma distribution, the following result is used.

If x_i is a random variable with density

$$f(x_i) = e^{-x_i}, \ 0 < x_i < \infty \quad \text{for } i = 1,2, \ldots, n \tag{4.2.1}$$

then the random variable

$$Y = \sum_{i=1}^{n} x_i \tag{4.2.2}$$

has the gamma density

$$g(y) = (1/\Gamma(n))y^{n-1} e^{-y}, \ 0 < y < \infty \tag{4.2.3}$$

when the x_i's are stochastically independent. Consequently, the generation of random variates from the gamma distribution involves only a simple

modification of the exponential method.

1) Using the method given in 4.1, generate n exponentially distributed random numbers $x_1, x_2, \ldots, x_n$.

2) Set $Y = \sum_{i=1}^{n} x_i$ (if a scale parameter $B \neq 1$ is desired, set $Y = B \cdot \sum_{i=1}^{n} x_i$).

4.3 Generation of Random Variates having the Standard Normal Distribution

Using the composition principle of 3.1, the absolute normal density

$$f(t) = \sqrt{2/\pi}\ e^{-\frac{1}{2}t^2},\ t \geq 0 \tag{4.3.1}$$

is represented as a mixture of three densities

$$f(t) = p_1 g_1(t) + p_2 g_2(t) + p_3 g_3(t). \tag{4.3.2}$$

Again, g_1 is a series of rectangles defined for $t \leq 3$, g_2 represents the toothlike regions above g_1 and below f defined for $t \leq 3$, and g_3 is the tail of the density (4.3.1) defined for $t \geq 3$.

Random variates from g_1 are generated by means of the technique given in 3.1. Define

$$M = 3/\Delta \quad \text{where } \Delta = 10^{-1} \text{(decimal machine)}$$
$$\text{or } \Delta = 16^{-1} \text{(binary machine), and} \tag{4.3.3}$$

discrete variables

$$t_i = (i - 1) \cdot \Delta \quad \text{for } i = 1, 2, \ldots, M. \tag{4.3.4}$$

Assign each t_i a probability

$$P_i = A \cdot f(t_{i+1}) = A \cdot \sqrt{2/\pi}\; e^{-\frac{1}{2}t_{i+1}^2} = .\delta_{1i}\delta_{2i}\ldots \tag{4.3.5}$$

Applying the storage technique described in Chapter 2, store the t_i's based on the high order 4 octal digits (binary machine) or the high order 3 decimal digits. Note that as in the exponential case $\delta_{1i} = 0$ for all i, and that the sets of discrete values are overlapped (see the A-table in the listing of GEN3 in the appendix). Now define values

$$Q_0 = 0, \quad Q_r = \beta^{-r} \sum_{i=1}^{M} \delta_{ri} \tag{4.3.6}$$

and

$$\Pi_0 = 0, \quad \Pi_r = \sum_{j=1}^{r} \sum_{i=1}^{M} \delta_{ji}; \tag{4.3.7}$$

also define

$$S_0 = 0, \quad S_k = \beta^k \cdot \sum_{r=1}^{k} Q_r \tag{4.3.8}$$

and

$$N_i = \Pi_{k-1} - \beta \cdot S_{k-1}, \tag{4.3.9}$$

for a binary machine r,k = 1,2,3,4 and $\beta = 8$ for a decimal machine, r,k = 1,2,3 and $\beta = 10$. The S-values and N-values are also recorded in the program listing of GEN3 (note that the N-values have been adjusted to allow for overlap between sets).

The procedure for the generation of random variates from this truncated portion of g_1 is: (binary case)

1) Generate a uniform (0,1) random number $u = .d_1d_2\ldots$.
2) If $d_1d_2 < S_2$, set $Y = A(d_1d_2 + N_2) + \Delta \cdot (.d_5d_6\ldots)$; otherwise go to (3).

3) If $d_1d_2d_3 < S_3$, set $Y = A\{d_1d_2d_3 + N_3\} + \Delta \cdot (.d_5d_6\ldots)$; otherwise go to (4).

4) If $d_1d_2d_3d_4 < S_4$, set $Y = A\{d_1d_2d_3d_4 + N_4\} + \Delta \cdot (.d_5d_6\ldots)$; otherwise generate a random variable from g_2, g_3, or the residual of g_1.

Random variates from the residual of g_1 and from g_2 are produced by first choosing one of the rectangles of g_1 with appropriate probability and then testing to determine whether the required variate should come from the rectangular residue of g_1 or from the toothlike region of g_2. As presented in 3.1, this is accomplished as follows:

In M consecutive locations store each t_i once, and for each t_i define a pair of values as in (3.1.12), (3.1.13), and (3.1.14):

$$C(1) = S_4 + R[A(1)], \tag{4.3.10}$$

$$B(k) = C(k) + T[A(k)], \tag{4.3.11}$$

$$C(k+1) = C(k) + R[A(k+1)]. \tag{4.3.12}$$

Note that in the listing of GEN3 this portion of the A-table occupies locations 690 thru 6BF. However, for convenience it is assumed that location 690 correspond to A(1). R[A(k)] and T[A(k)] denote respectively the area of the rectangular remnant and the area of the toothlike region corresponding to the t-value occupying location A(k). For the normal case:

$$R[A(k)] = P[A(k)] - .\delta[1,A(k)]\delta[2,A(k)]\delta[3,A(k)]\delta[4,A(k)] \tag{4.3.13}$$

where $P[A(k)]$ is defined in (4.3.5) as

$$P[A(k)] = \Delta \cdot f(A(k) + \Delta) = \Delta \cdot \sqrt{2/\pi} \cdot e^{-\frac{1}{2}(A(k)+\Delta)^2}$$

$$= .\delta[1,A(k)]\delta[2,A(k)]\ldots, \qquad (4.3.14)$$

and

$$T[A(k)] = F(A(k) + \Delta) - F(A(k)) - P[A(k)] \qquad (4.3.15)$$

where

$$F(x) = \sqrt{2/\pi} \int_0^x e^{-\frac{1}{2}t^2} dt \qquad (4.3.16)$$

Once having determined whether a variate with a rectangular density (g_1) or a variate with a toothlike density (g_2) is required, the schemes given in 3.1 and in 3.5 are used to produce the variate as required. This procedure is summarized as follows:

1) Let u be a uniform (0,1) random number such that $S_4 \leq u < p_1 + p_2 = F(3)$.

2) Test $u < B(k)$ for $k = 1,2, \ldots, M$ until the inequality is satisfied for some $k = k'$.

3) If $u < C(k')$, then generate a new uniform (0,1) random number v and put $Y = A(k') + \Delta \cdot v$ (this produces a variate from g_1); otherwise go to (4).

4) Generate independent uniform (0,1) random numbers u_1 and u_2.

5) If $\max(u_1,u_2) \leq D(k')^*$, set $Y = A(k') + \Delta \cdot \min(u_1,u_2)$. This produces a variate from g_2 as described in 3.5; otherwise go to (6).

6) Set $W = -\frac{1}{2}(\Delta \cdot \min(u_1,u_2) - \Delta) \cdot [2 \cdot A(k') + \Delta \cdot \min(u_1,u_2) + \Delta]$

---

* The $D(k)$'s are tabled values of the a_i/b_i ratios given in (3.5.12) and in (3.5.14) corresponding to the t_i stored in $A(k')$.

and test $|u_1 - u_2| < E(k') \cdot (e^W - 1)$**. If yes, set

$Y = A(k') + \Delta \cdot \min(u_1,u_2)$. If no go to step (4) and try again.

A random variate from the density g_3 is produced by the technique presented in 3.6. An absolute normal variable $|Y|$, conditioned by $|Y| > 3$, is required. The procedure is summarized as follows:

1) Generate pairs of uniform (0,1) random numbers u_1 and u_2 until the condition $u_1^2 + u_2^2 \leq 1$ is satisfied.

2) Form pairs x_1 and x_2 as described in (3.6.1) and (3.6.2)

$$x_1 = u_1 \cdot [\{9 - 2\ln(u_1^2 + u_2^2)\}/\{u_1^2 + u_2^2\}]^{\frac{1}{2}}$$

$$x_2 = u_2 \cdot [\{9 - 2\ln(u_1^2 + u_2^2)\}/\{u_1^2 + u_2^2\}]^{\frac{1}{2}}$$

3) Test $x_1 > 3$. If yes, set $Y = x_1$. If no, test $x_2 > 3$. If yes, put $Y = x_2$. If no, go to step (1) and repeat the procedure.

In [12] Marsaglia presents a schematic and the required constants for the generation of random variates from the absolute normal distribution. This method results in random variates having the density (4.3.1). In order to obtain random variates from the standard normal distribution.

$$f(t) = (1/\sqrt{2\pi})\ e^{-\frac{1}{2}t^2}, \ -\infty < t < \infty \ , \qquad (4.3.17)$$

a random + or - sign must be attached at some point in the procedure. In GEN3 this is done by generating a uniform (0,1) random number u and testing $u < \frac{1}{2}$. If yes, a - sign is affixed to the variate; otherwise a positive variate is returned.

The procedures presented in this chapter derive their speed from the

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** This expression is simply an alternative way of testing $b_i |u-v| \leq f(A(k') + \Delta \cdot \min(u_1,u_2))$.

fact that the vast majority of variates generated come from the truncated portion of g_1. It requires only the generation of a uniform (0,1) random number, a series of compare instructions, and a table lookup. While the generation of variates from g_2 and g_3 may be somewhat time consuming, such variates are required only rarely. Consequently, that average time per generation is quite fast. As is evident these methods are exact. Precision is limited only by the word size of the computer.

CHAPTER 5

RESULTS OF TESTS PERFORMED ON

SEQUENCES OF PSEUDO-RANDOM NUMBERS

5.1 Random Numbers From The Uniform (0,1) Distribution

The tests performed on sequences of independent uniform (0,1) random numbers were suggested by MacLaren and Marsaglia in [9]. The stringency of these tests is justified in that the procedures for the generation of random variates from other distributions depend heavily upon the method used to obtain uniform (0,1) random numbers.

The tests made were chi-square (χ^2) goodness of fit tests on the distribution of the random numbers, pairs of random numbers, triples of random numbers, and the maximum and minimum of n random numbers. In general a sequence of ℓ variables $Y_1, Y_2, \ldots, Y_\ell$ was calculated from the sequence of random uniform variates. The range of the Y_i was divided into m cells of equal probability, p, and the number of occurrences, O_i, in each cell counted. The χ^2 statistic

$$\chi^2 = \sum_{i=1}^{m} (O_i - \ell \cdot p)^2/\ell \cdot p \qquad (5.1.1)$$

with m - 1 degrees of freedom was calculated and transformed to a standard normal deviate

$$T = (2 \cdot \chi^2)^{\frac{1}{2}} - (2 \cdot (m - 1) - 1)^{\frac{1}{2}} \qquad (5.1.2)$$

for each test. This form is valid for $m \geq 31$. The significance level of this normal deviate was then computed as

$$S = \int_0^T (1/\sqrt{2\pi})\ e^{-\frac{1}{2}t^2} dt\ , \tag{5.1.3}$$

In each test a total of 100,000 uniform numbers was generated. For convenience the numbers were generated in 10,000 sets of 10 uniform numbers each. Five separate runs were made and the tests performed as follows:

1) Uniformity. The unit interval was divided into 1000 segments. Each segment has an expected value of 100 occurrences. The χ^2 statistic was computed as

$$\chi^2 = \sum_{i=1}^{1000} (O_i - 100)^2/100 \tag{5.1.4}$$

where O_i is the number of observed occurrences in the i^{th} segment.

2) Pairs. Successive pairs of uniform numbers were taken as the coordinates of a point in the unit square. The unit square was partitioned into 100 cells, each with an expected value of 500 occurrences. The χ^2 statistic was computed as

$$\chi^2 = \sum_{i=1}^{10} \sum_{j=1}^{10} (O_{ij} - 500)^2/500 \tag{5.1.5}$$

where O_{ij} is the number of observed occurrences in the ij^{th} cell.

3) Triples. Successive triples of uniform numbers were taken as the coordinates of a point in the unit cube (every tenth number was skipped). The unit cube was partitioned into 1000 cells with an expected

value of 30 occurrences in each cell. The χ^2 statistic was computed as

$$\chi^2 = \sum_{i=1}^{10} \sum_{j=1}^{10} \sum_{k=1}^{10} (O_{ijk} - 30)^2/30 \tag{5.1.6}$$

where O_{ijk} is the number of observed occurrences in the ijk^{th} cell.

4) Maximum of n random values. If $u_1, u_2, \ldots, u_n$ are independent uniform (0,1) deviates, then

$$W = \max(u_1, u_2, \ldots, u_n) \tag{5.1.7}$$

should have the distribution

$$P(W \le a) = F(a) = a^n \quad \text{for } 0 < a < 1, \text{ and} \tag{5.1.8}$$

$$F(W) = [\max(u_1, u_2, \ldots, u_n)]^n \tag{5.1.9}$$

should be uniformly distributed over the interval (0,1). A total of 10,000 W's were generated for each n, and F(W) was tested for uniformity using the χ^2 goodness of fit test for 100 equal subintervals. The χ^2 statistic was computed as

$$\chi^2 = \sum_{i=1}^{100} (O_i - 100)^2/100 \tag{5.1.10}$$

where O_i is the number of observed occurrences in the i^{th} subinterval. In this test n takes the values 2 and 5.

5) Minimum of n random values. This test is the same as that for the maximum of n except

$$W = \min(u_1, u_2, \ldots, u_n) \text{ and} \tag{5.1.11}$$

and

$$F(W) = 1 - (1 - W)^n \tag{5.1.12}$$

which should be uniformly distributed over the unit interval. The 10,000 W's were segmented into 100 equal subintervals and the χ^2 test computed as in (5.1.10). In this test n takes the values 3 and 10.

The results of these tests are reported in Table 5.1.1. The results compare favorably with those presented in [9] and justify the use of this particular uniform (0,1) random number generator.

TABLE 5.1.1

RUN #1

TEST	CHI-SQUARE	D.F.	SIGNIFICANCE
UNIFORMITY	975.400	999	0.30186688
PAIRS	89.6720	99	0.25987495
TRIPLES	1034.7333	999	0.78918171
MAX OF 2	97.1800	99	0.46241028
MAX OF 5	80.7800	99	0.09257838
MIN OF 3	109.5200	99	0.77766504
MIN OF 10	75.0400	99	0.03713434

RUN #2

TEST	CHI-SQUARE	D.F.	SIGNIFICANCE
UNIFORMITY	1047.1600	999	0.85902187
PAIRS	92.8440	99	0.34129536
TRIPLES	1022.4667	999	0.70302930
MAX OF 2	104.4800	99	0.66267689
MAX OF 5	96.9800	99	0.45671365
MIN OF 3	102.1000	99	0.60032472
MIN OF 10	97.7400	99	0.47836695

TABLE 5.1.1 (continued)

RUN #3

TEST	CHI-SQUARE	D.F.	SIGNIFICANCE
UNIFORMITY	1006.3400	999	0.56949803
PAIRS	93.8720	99	0.36930366
TRIPLES	1052.0000	999	0.88131258
MAX OF 2	93.5400	99	0.36019046
MAX OF 5	104.5000	99	0.66318213
MIN OF 3	102.7400	99	0.61749364
MIN OF 10	107.0600	99	0.72480337

RUN #4

TEST	CHI-SQUARE	D.F.	SIGNIFICANCE
UNIFORMITY	970.2600	999	0.26223433
PAIRS	103.9040	99	0.64798213
TRIPLES	1084.4667	999	0.97021768
MAX OF 2	83.2400	99	0.12861638
MAX OF 5	96.4200	99	0.44078290
MIN OF 3	76.7000	99	0.04945168
MIN OF 10	116.5000	99	0.89040197

TABLE 5.1.1 (continued)

RUN #5

TEST	CHI-SQUARE	D.F.	SIGNIFICANCE
UNIFORMITY	1052.4000	999	0.88303445
PAIRS	82.5080	99	0.11705985
TRIPLES	1013.5333	999	0.63124447
MAX OF 2	92.6200	99	0.33528034
MAX OF 5	98.8400	99	0.50965471
MIN OF 3	140.6600	99	0.99687357
MIN OF 10	136.3000	99	0.99333696

5.2 Random Numbers Having A Discrete Distribution

Sequences of random numbers from the binomial density

$$f(x) = \binom{N}{x} \cdot p^x \cdot (1 - p)^{N-x}, \ 0 < p < 1, \quad x=0,1,\ldots,N \qquad (5.2.1)$$

were generated for a variety of parameter combinations. The range of the variable was divided into k intervals of probability q_i, such that the expected value of each interval, $m \cdot q_i$, was at least 10. The χ^2 statistic was computed as

$$\chi^2 = \sum_{i=1}^{k} (O_i - m \cdot q_i)^2/m \cdot q_i \qquad (5.2.2)$$

where O_i is the number of observed occurrences in the i^{th} interval, and m is the size of the sample generated. The significance level was computed as

$$S = (1/\Gamma(\nu/2)) \cdot \int_0^{\chi^2/2} e^{-t} \cdot t^{(\nu/2 - 1)} dt \qquad (5.2.3)$$

where ν = degrees of freedom, in this case $\nu = k-1$. The results of these tests are recorded in table 5.2.1. A sample size of 1000 random numbers were generated for each test.

Random numbers from the Poisson density

$$f(x) = \lambda^x e^{-\lambda}/x! \ , \ 0 < \lambda < \infty \ , \ x = 0,1,\ldots,\infty \qquad (5.2.4)$$

and from the negative binomial density

$$f(x) = \binom{r+x-1}{x} \cdot p^r \cdot (1-p)^x, \ r \geq 0, \ \ 0 < p < 1, \qquad (5.2.5)$$

$$x = 0,1,2,\ldots,\infty;$$

were tested in the same manner as random numbers from the binomial density (5.2.1). The results of these tests are recorded in table 5.2.2 and 5.2.3 respectively.

TABLE 5.2.1

RESULTS OF CHI-SQUARE GOODNESS OF FIT TESTS
ON RANDOM NUMBERS FROM THE BINOMIAL DENSITY

P	N	D.F.	CHI-SQUARE	SIGNIFICANCE
0.100	10	4	6.03494	0.8034458581
0.100	20	6	6.44102	0.6243600635
0.100	30	7	2.52631	0.0748969943
0.100	40	9	6.33123	0.2936355148
0.100	50	9	12.92191	0.8338284965
0.200	10	5	7.56431	0.8180614798
0.200	20	8	16.34847	0.9623445588
0.200	30	10	15.13733	0.8728654417
0.200	40	11	9.00770	0.3788180126
0.200	50	13	12.00444	0.4727203102
0.300	10	7	7.97911	0.6655606411
0.300	20	9	10.98540	0.7232888983
0.300	30	11	17.78713	0.9133467551
0.300	40	13	12.08372	0.4792085395
0.300	50	15	21.66071	0.8829858823
0.400	10	7	7.29406	0.6010818180
0.400	20	10	12.18194	0.7269356404
0.400	30	12	20.64248	0.9441325898
0.400	40	14	10.05124	0.2415723570
0.400	50	16	12.35961	0.2811236434
0.500	10	8	6.86156	0.4483592431
0.500	20	10	12.57912	0.7518355135
0.500	30	12	10.41700	0.4205727006
0.500	40	14	23.28293	0.9441810635
0.500	50	16	9.42171	0.1049637735
0.600	10	7	6.92477	0.5632454466
0.600	20	10	18.27415	0.9494885520
0.600	30	12	10.17474	0.3993657394
0.600	40	14	11.25580	0.3341658613
0.600	50	16	10.03645	0.1352818684

TABLE 5.2.1 (continued)

P	N	D.F.	CHI-SQUARE	SIGNIFICANCE
0.700	10	7	3.47126	0.1617396643
0.700	20	9	10.70221	0.7033259190
0.700	30	11	12.02199	0.6380075901
0.700	40	13	13.88283	0.6178529373
0.700	50	15	18.60988	0.7680404078
0.800	10	5	4.53985	0.5254310881
0.800	20	8	16.74590	0.9671335640
0.800	30	10	10.97150	0.6402573630
0.800	40	11	9.02074	0.3800224541
0.800	50	13	15.05620	0.6961423824
0.900	10	4	3.31137	0.4928666476
0.900	20	6	4.40204	0.3775593406
0.900	30	7	6.78871	0.5487935778
0.900	40	9	6.74965	0.3368342656
0.900	50	9	5.04256	0.1694175512

RESULTS OF CHI-SQUARE GOODNESS OF FIT TESTS ON
RANDOM NUMBERS FROM THE POISSON DENSITY.

LAM	D.F.	CHI-SQUARE	SIGNIFICANCE
5.00	10	13.57652	0.8068012578
10.00	15	8.51890	0.0986988385
15.00	17	16.33853	0.5000245564
20.00	20	18.35403	0.4359007928
25.00	21	28.70494	0.8787325544
30.00	23	24.94425	0.6468336304
35.00	25	26.91360	0.6397852204
40.00	27	43.19749	0.9750174127
45.00	28	22.55633	0.2450911949
50.00	30	23.26187	0.1956567510
55.00	30	42.01434	0.9286283430
60.00	31	34.81330	0.7087406320
65.00	33	29.49011	0.3573849816
70.00	34	29.73472	0.3231975827
75.00	35	26.18810	0.1410044103
80.00	35	26.80832	0.1619395204
85.00	36	21.61020	0.0278060381
90.00	37	28.29852	0.1527661009
95.00	38	30.51639	0.1992224404
100.00	39	37.10850	0.4435994584

TABLE 5.2.2

TABLE 5.2.3

RESULTS OF CHI-SQUARE GOODNESS OF FIT TESTS ON RANDOM NUMBERS FROM THE NEGATIVE BINOMIAL DENSITY.

P	R	D.F.	CHI-SQUARE	SIGNIFICANCE
0.100	2	41	36.08627	0.3115372475
0.100	4	57	51.64328	0.3244292796
0.100	6	67	77.44254	0.8201664431
0.100	8	72	54.21390	0.0583581269
0.100	10	76	58.52535	0.0683503608
0.200	2	23	20.00081	0.3581367902
0.200	4	33	27.73007	0.2730820886
0.200	6	40	43.00986	0.6563763897
0.200	8	45	42.83166	0.4357673160
0.200	10	49	38.69528	0.1455417220
0.300	2	15	11.10495	0.2548804445
0.300	4	22	25.94653	0.7460151638
0.300	6	28	17.46899	0.0612580512
0.300	8	31	25.10221	0.2368731801
0.300	10	34	18.65952	0.0151991805
0.400	2	11	12.32864	0.6605459138
0.400	4	16	7.20666	0.0309309696
0.400	6	20	20.18198	0.5534007984
0.400	8	23	23.12594	0.5466209213
0.400	10	25	22.04679	0.3669595654
0.500	2	8	4.96920	0.2391366628
0.500	4	12	13.04976	0.6345655810
0.500	6	15	16.03409	0.6202105433
0.500	8	17	25.16863	0.9090016643
0.500	10	19	24.55579	0.8243107298
0.600	2	6	13.11971	0.9588261108
0.600	4	9	14.42509	0.8920130256
0.600	6	11	13.29194	0.7253265963
0.600	8	13	17.84469	0.8364937782

TABLE 5.2.3 (continued)

P	R	D.F.	CHI-SQUARE	SIGNIFICANCE
0.600	10	14	13.95647	0.5470410000
0.700	2	5	6.42472	0.7329438096
0.700	4	7	11.10162	0.8657526004
0.700	6	8	9.98931	0.7342226732
0.700	8	10	4.82568	0.0974859792
0.700	10	11	14.96953	0.8161040822
0.800	2	3	1.03482	0.2071720727
0.800	4	5	2.88184	0.2818034287
0.800	6	6	1.48994	0.0398398009
0.800	8	7	4.59512	0.2907654804
0.800	10	8	2.13612	0.0234479888
0.900	2	2	0.67942	0.2880246110
0.900	4	3	0.23420	0.0281120703
0.900	6	3	4.99992	0.8281970919
0.900	8	4	2.59324	0.3719796012
0.900	10	4	2.34658	0.3276973945

5.3 Random Numbers From Continuous Distribution Functions

Sequences of random numbers from the exponential

$$f(t) = e^{-t}, \quad 0 < t < \infty \tag{5.3.10}$$

and normal

$$f(x) = (1/\sqrt{2\pi})\ e^{-\frac{1}{2}x}, \quad -\infty < x < \infty \tag{5.3.2}$$

densities were generated, and segmented into 20 intervals of approximately equal probability, $p_i = 0.05$. Samples of $m = 1000$ random numbers were generated and the χ^2 statistic computed as

$$\chi^2 = \sum_{i=1}^{20} (O_i - m \cdot p_i)^2/m \cdot p_i \tag{5.3.3}$$

where O_i is the number of observed occurrences in the i^{th} interval. The significance level was calculated as given in (5.2.3) with $\nu = 19$. The results of these tests are given in tables 5.3.1 and 5.3.2.

Sequences of random numbers from the gamma density

$$f(t) = (1/\Gamma(n))\ t^{n-1}\ e^{-t}, \quad 0 < t < \infty, \tag{5.3.4}$$

were tested in a somewhat different manner. If t is a variable with density (5.3.4) then

$$y = 2 \cdot (t)^{\frac{1}{2}} - (4 \cdot n - 1)^{\frac{1}{2}} \tag{5.3.5}$$

should have an approximate standard normal density when n is large. Sequences of m gamma distributed random numbers were generated and using the transformation (5.3.5), transformed to m approximately standard normal variables. As before, the normal distribution was partitioned into 20

RESULTS OF CHI-SQUARE GOODNESS OF FIT TESTS ON RANDOM NUMBERS FROM THE EXPONENTIAL DENSITY, F(X)=EXP(-X).

D.F.	CHI-SQUARE	SIGNIFICANCE
19	20.76000	0.6497876
19	11.24000	0.0844989
19	17.64000	0.4534074
19	22.68000	0.7482694
19	13.68000	0.1979842
19	18.52000	0.5120074
19	11.48000	0.0933650
19	17.36000	0.4345072
19	24.24000	0.8128932
19	37.24000	0.9925960

TABLE 5.3.1

RESULTS OF CHI-SQUARE GOODNESS OF FIT TESTS ON RANDOM NUMBERS FROM THE NORMAL DENSITY

D.F.	CHI-SQUARE	SIGNIFICANCE
19	19.59699	0.5808149783
19	25.42150	0.8528693863
19	18.29836	0.4974018037
19	6.71247	0.0044039802
19	14.75770	0.2621316817
19	19.91207	0.6001134403
19	20.76446	0.6500395713
19	23.50146	0.7840244000
19	11.96570	0.1129139927
19	18.02468	0.4792096579

TABLE 5.3.2

intervals of approximately equal probability $p_i = 0.05$, and the χ^2 statistic computed as before. The results of these tests are recorded in table 5.3.3.

RESULTS OF CHI-SQUARE GOODNESS OF FIT TESTS ON TRANSFORMED GAMMA RANDOM NUMBERS

SAMPLE SIZE	N	D.F.	CHI-SQUARE	SIGNIFICANCE
100	50	19	12.30156	0.1276659399
100	100	19	14.27597	0.2326276646
100	200	19	13.79290	0.2043727108
200	50	19	13.99520	0.2160288470
200	100	19	20.25710	0.6207407158
200	200	19	23.61244	0.7885588803
500	50	19	23.13291	0.7684615439
500	100	19	21.91446	0.7114659110
500	200	19	32.02104	0.9689153752

TABLE 5.3.3

5.4 Discussion of Results.

It should be emphasized that no test or series of tests can assure the suitability of a given sequence of random numbers for a particular problem. When possible the sequence should be tested on a similar problem with known solution. However, the results presented here establish the fundamental reliability of the generation procedures presented in this paper. The significance levels obtained should follow the uniform distribution on (0,1) if the corresponding null hypothesis tested are valid. The above results are in agreement with this condition.

CHAPTER 6
DESCRIPTION AND USE OF GENERATOR PROGRAMS

6.1 GENO

GENO supplies the user with a very fast procedure for the generation of independent uniform (0,1) random numbers with density

$$f(u) = 1, \quad 0 < u < 1. \tag{6.1.1}$$

The method used was suggested by Dr. Rolf Bargmann [1] and is a form of the multiplicative congruential procedure,

$$u_{n+1} = a \cdot u_n \bmod 2^{32} \tag{6.1.2}$$

where $a = 5^{13}$. This procedure requires that u_0, the starting value, be an odd positive integer.

GENO is written as a subroutine with entry point URAN. It is called as follows:

$$U = URAN(IOD), \tag{6.1.3}$$

where "IOD" is the starting value and must be an odd positive integer. The desired random number U in (6.1.3), is returned to the calling program in single precision real mode.

GENO requires 20 words (80 bytes) of storage space and an average time per generation of 25 usec.

6.2 GEN1

GEN1 provides the user with a fast procedure for the generation of exponentially distributed random numbers

$$f(x) = e^{-x}, \quad 0 < x < \infty. \qquad (6.2.1)$$

The method used is that described in 4.1, and presented by Marsaglia in [13].

GEN1 is written as a subroutine with entry point RANEXP. It is called from a FORTRAN main program as follows:

$$X = RANEXP(IOD). \qquad (6.2.2)$$

The parameter "IOD" primes the scheme for the generation of uniform (0,1) random numbers, "IOD" must be an odd positive integer to insure the proper generation of u. The required random number x in (6.2.2) is returned to the calling program in single precision real mode.

The average time per generation is approximately 70 usec. Memory requirement for GEN1 is 307 words (1228 bytes).

6.3 GEN2

GEN2 supplies the user with a fast procedure for the generation of gamma ($\alpha = n$, $\beta = 1$)

$$g(y) = 1/\Gamma(n) \cdot y^{n-1} \cdot e^{-y}, \quad 0 < y < \infty \qquad (6.3.1)$$

distributed random numbers. The technique used is described in 4.2.

GEN2 is written as a subroutine with entry point RANGAM, and is called from a FORTRAN main program as follows:

$$Y = \text{RANGAM(IOD,N)}. \qquad (6.3.2)$$

As before, "IOD" must be an odd positive integer and n = "N" in (6.3.1). The required variate is returned in single precision real mode.

GEN2 requires 313 words of memory space and approximately 65 · N usec per generation for N > 1.

6.4 GEN3

GEN3 supplies the user with a fast procedure for the generation of random numbers from the standard normal density

$$f(x) = (1/\sqrt{2\pi})\ e^{-\frac{1}{2}x^2}, \ -\infty < x < \infty. \qquad (6.4.1)$$

The procedure is described in 4.3 and by Marsaglia in [12].

GEN3 is written as a subroutine and is called from a FORTRAN main program as follows:

$$X = \text{RANORM(IOD)}. \qquad (6.4.2)$$

The parameter "IOD" must be an odd positive integer, and the required variate X in (6.4.2) is returned to the calling program in single precision real mode.

Memory requirement for GEN3 is 442 words. The average time per generation is approximately 65 usec.

6.5 GEN4

GEN4 provides a fast procedure for the generation of random numbers from the binomial distribution

$$f(x) = \binom{n}{x} p^x(1 - p)^{n-x}, \ 0 < p < 1, \ x=0,1,\ldots,n. \qquad (6.5.1)$$

GEN4 is written as a subroutine with two entry points, BSETUP and RANBI. The user calls BSETUP with the parameters "P" and "N" e.g.

$$\text{CALL BSETUP(P,N)}. \tag{6.5.2}$$

BSETUP calculates point probabilities using the recursion relation

$$f(0) = (1 - P)^N \text{ and} \tag{6.4.3}$$

$$f(x+1) = f(x) \cdot P \cdot (N-x)/((x+1) \cdot (1-P)).$$

The four sets of discrete values are then stored using the procedure described in Chapter 2. BSETUP need be called only once for a given "P" and "N", but must be called at least once prior to calling RANBI.

RANBI addresses the very fast scheme that generates the required variates. It is called as follows:

$$X = \text{RANBI(IOD)} \tag{6.4.4}$$

where "IOD" must be an odd positive integer, and X is returned in single precision real mode.

In its present form GEN4 requires the "P" be in the real mode, $0 < P < 1$, and that "N" be in the integer mode and not exceed 255. The program requires 948 words of memory and an average time per generation of approximately 35-40 usec. Figure 2.1 describes the flowchart of GEN4 except for the section that computes point probabilities.

If the user desires random numbers returned in the integer mode, he need only delete

```
STMT
199     ST      0, RES
200     MVI     RES, X'46'
201     AE      0, RES
```

and change the designation RANBI to IANBI in his calling statement and in statements

170	ENTRY RANBI	
171	USING RANBI, 15	
173	DC	CL7' ..RANBI'
174RANBI	STM	1,3, 24(13)

Storage requirements may also be altered simply by changing

STMT		
146	C	7, = F'2000'
222	DS	2000XL1.

6.5 GEN5

GEN5 supplies a fast routine for the generation of random numbers from the poisson density

$$f(x) = \lambda^x e^{-\lambda}/x! \ , \ 0 < \lambda < \infty \ , \ x = 0,1,.... \qquad (6.5.1)$$

Since the domain of x is infinite, it is neccessary to truncate the distribution. This is done at both low and high point values whenever a point probability is less than 16^{-4}.

GEN5 is written as a subroutine with two entry points, PSETUP and RANPOI. The user calls PSETUP with the parameter λ, e.g.

$$\text{CALL PSETUP(ALAM).} \qquad (6.5.2)$$

PSETUP calculates point probabilities using the recursion relation

$$f(0) = e^{-\lambda}$$

$$f(x+1) = f(x) \cdot \lambda/(x+1), \qquad (6.5.3)$$

and stores the four sets of discrete values by means of the technique described in Chapter 2. PSETUP need be called only once for a given λ, but must be called at least once before the initial call to RANPOI.

The entry point RANPOI addresses the very fast scheme that performs the actual generation of the required variate. It is called as follows:

$$X = \text{RANPOI(IOD)}, \tag{6.5.4}$$

where "IOD" must be an odd positive integer and the desired random number X is returned in single precision real mode.

GEN5 requires 972 words of memory space and an average time per generation of approximately 35-40 usec. The flow of GEN5 follows that presented in figure 2.1 except for the section that computes the point probabilities.

If the user desires his result returned in the integer mode, he need only delete

```
STMT
 190          ST          0,RES
 191          MVI         RES, X'46'
 192          AE          0,RES
```

and change the designation RANPOI to IANPOI in his calling statement and in statements

```
STMT
 161                         ENTRY  RANPOI
 162                         USING  RANPOI
 164                         DC     CL7'   RANPOI'
 165 RANPOI                  STM    1,3,24(13)
```

Storage requirement may also be altered by changing

```
STMT

 137                        C     7, = F'2000'

 211  A                     DS    2000XL1
```

6.6 GEN6

GEN6 provides a fast routine for generating random numbers from the negative binomial density

$$f(x) = \binom{x+r-1}{x} p^r (1-p)^x \quad , 0 < p < 1, \ x = 0,1, \ldots, \qquad (6.6.1)$$

$$\text{and } r \geq 0.$$

The domain of x is infinite, consequently, the distribution is truncated at both low and high point values whenever a point probability is less than 16^{-4}.

GEN6 is written as a subroutine with two entry points, NBSETU and RANEBI. NBSETU receives the parameters, p and r, and calculates point probabilities using the recursion relation

$$f(0) = p^r \qquad (6.6.2)$$

$$f(x+1) = f(x) \cdot (1-p) \cdot (x+r)/(x+1).$$

Having computed the point probabilities, NBSETU then stores the four sets of discrete values using the procedure described in Chapter 2. NBSETU is called as follows:

$$\text{CALL NBSETU(P,IR)}, \qquad (6.6.3)$$

where "IR" must be in the integer mode and 0 < "P" < 1. NBSETU need be

called only once for a given "P" and "IR", but must be called at least once prior to the initial call to RANEBI.

RANEBI addresses the very fast scheme for the generation of the required variates, and is called as follows:

$$X = \text{RANEBI(IOD)}, \tag{6.6.4}$$

where "IOD", the primer, must be an odd positive integer, and X is returned in single precision real mode.

GEN6 requires 944 words of storage space and an average time per generation in the 35-40 usec range. The flow-chart of GEN6 follows that of figure 2.1 except for the section of GEN6 that calculates the point probabilities.

6.7 GEN7

GEN7 provides a fast procedure for generating discrete random variables with any specified probability distribution denoted by a vector $P = \{p_1, p_2, \ldots\}$. It is written as a subroutine with two entry points, DSETUP and RANDIS. DSETUP is called with 3 parameters, "P", "IX", "N"; where "P" is a vector of probabilities (real mode), "IX" is a vector of corresponding discrete values (integer mode), and "N" is the number of elements in vectors, e.g.

$$\text{CALL DSETUP(P,IX,N)}. \tag{6.7.1}$$

The discrete values of the "IX" vector are stored in four sets based on the high order 4 digits of the elements of the "P" vector. This procedure is given in Chapter 2 and illustrated in figure 2.1. DSETUP need be called

only once for a particular set of parameters, however, it must be called at least once prior to the initial call to RANDIS.

RANDIS addresses the very fast scheme that performs the actual generation of the required variates. It is called as follows:

$$X = \text{RANDIS(IOD)}, \tag{6.7.2}$$

where "IOD" must be an odd positive integer and "X", the generated variate is returned in single precision real mode.

In its present form GEN7 requires that the elements of the "IX" vector be in the integer mode and that no value exceed 255. Also, the parameter "N", the number of elements in the vectors, may not exceed 256. If the user wishes his discrete variables, members of the "IX" vector, to be in the real mode or to be in the integer mode but with values greater than 255, he need only make the changes indicated by table 6.1. If he desires to have his result returned in the integer mode he need only delete the following

```
STMT
 149           ST          0,RES
 150           MVI         RES, X'46'
 151           AE          0,RES,
```

and change the designation RANDIS to IANDIS in his calling statement and in the following

```
STMT
 120           ENTRY       RANDIS
 121           USING       RANDIS, 15
 123           DC          CL7'  RANDIS'
 124 RANDIS    STM         1,3,24(13).
```

Storage requirement may also be altered simply by changing

STMT		
97	C	7, = F'2000'
168 A	DS	2000XL1

In its present form GEN7 requires 904 words of memory space and 35-40 usec per generation.

STMT			INTEGER > 255		REAL MODE	
87	L	6, = F'1'	LA	6,4	LA	6,4
97	C	7, = F'2000'	C	7, = F'SPAC'	C	7, = F'SPAC'
99	L	2, 0(3,11)	L	2, 0(3,11)	LE	2, 0(3,11)
101	STC	2, A(12)	ST	2, A(12)	STE	2, A(12)
106	CR	0,4	C	0, = F'16'	C	0, = F'16'
148 EX	IC	0, A(2)	EX SLL	2, 2	EX SLL	2, 2
168 A	DS	2000 XL1	A DS	SPAC XL4	A DS	SPAC XL4

INTEGER > 255:

ALSO INSERT BETWEEN
STMT 95 AND STMT 96
SLL 8, 2

REAL MODE:

ALSO INSERT BETWEEN
STMT 95 AND STMT 96
SLL 8, 2
AND DELETE STMT 149
THROUGH STMT 151.

TABLE 6.1

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APPENDIX A

Assembler Listing of GENO

NOT REPRODUCIBLE

```
 LOC  OBJECT CODE    ADDR1 ADDR2  STMT   SOURCE STATEMENT                                                   F38APR71)   6/30/71

000000                               1 GENO      START 0
                                     2 *********************************************************************
                                     3 * SOURCE---L.E.CANNON, UGA, DEPT. OF STAT., APR 1970                *
                                     4 * PURPOSE--TO SUPPLY THE USER WITH A FAST PROCEDURE FOR GENERATING UN-*
                                     5 *          IFORM(0,1) RANDOM NUMBERS.                               *
                                     6 * USAGE----X=URAN(IOD). 'IOD' MUST BE AN ODD POSITIVE INTEGER. THE RE-*
                                     7 *          QUIRED VARIATE IS RETURNED IN SINGLE PRECISION REAL MODE.  *
                                     8 * EXAMPLE:X=URAN(56865)  'X' WILL BE UNIFORM ON (0,1).              *
                                     9 * METHOD---THE METHOD IS THE MULTIPLICATIVE CONGRUENTIAL PROCEDURE    *
                                    10 *                                X(I+1)=X(I)*A                      *
                                    11 *          WHERE A=5**13, AS SUGGESTED BY DR. R. F. BARGMANN.       *
                                    12 *********************************************************************
                                    13           ENTRY URAN
000008                              14           USING URAN,15
000000 07                           15           DC    XL1'07'
000001 404040E4D9C105               16           DC    CL7'   URAN'
000008 9013 D018           00018    17 URAN      STM   1,3,24(13)
00000C 3800                         18           SER   0,0          CLEAR FPR0 FOR RESULT.
00000E 47F0 F028           00030    19 BRANCH    B     TEMP         BRANCH TO TEMP.
000012 5C20 F044           0004C    20           M     2,RPAT       GENERATE U IN R3.
000016 5030 F048           00050    21           ST    3,LA         STORE U FOR NEXT CALL.
00001A 8830 0008           00008    22           SRL   3,8          CONVERT TO FLOATING POINT.
00001E 5030 F040           00048    23           ST    3,RES
000022 9240 F040     00048          24           MVI   RES,X'40'
000026 7A00 F040           00048    25           AE    0,RES        NORMALIZED RESULT IN FPR0.
00002A 9813 D018           00018    26           LM    1,3,24(13)
00002E 07FE                         27           BCR   15,14        RETURN TO CALLING PROGRAM.
000030 5830 F048           00050    28 TEMP      L     3,LA
000034 D203 F006 F028 0000E 00030   29           MVC   BRANCH,TEMP  REPLACE INSTR. AT BRANCH BY INSTR. AT TEMP.
00003A 5830 1000           00000    30           L     3,0(,1)      LOAD ADDRESS OF PRIMER.
00003E 5830 3000           00000    31           L     3,0(,3)      LOAD PRIMER.
000042 47F0 F00A           00012    32           B     BRANCH+4     BRANCH TO INSTR. FOLLOWING BRANCH.
000048                              33 RES       DS    1F
00004C 48C27395                     34 RPAT      DC    XL4'48C27395'
000050                              35 LA        DS    1F
                                    36           END
```

APPENDIX B

Assembler Listing of GEN1

```
                                                                                                        PAGE    1
  LOC  OBJECT CODE    ADDR1 ADDR2  STMT   SOURCE STATEMENT                                   FORAPR70   6/30/71
000000                              1 GEN1     START 0
                                    2 ****************************************************************
                                    3 * SOURCE---L.E.CANNON, UGA, DEPT. OF STAT., APR 1970          *
                                    4 * PURPOSE--TO SUPPLY THE USER WITH A FAST PROCEDURE FOR GENERATING EX-*
                                    5 *           PONENTIAL DISTRIBUTED RANDOM NUMBERS.               *
                                    6 * USAGE----X=RANEXP(I). 'I' MUST BE AN ODD INTEGER. IT PRIMES THE GEN-*
                                    7 *           ERATING SCHEME.                                     *
                                    8 * EXAMPLE--X=RANEXP(55695).                                     *
                                    9 * METHOD---THE METHOD IS THAT OF MARSAGLIA AS DESCRIBED IN 'COMMUNI- *
                                   10 *           CATIONS OF ACM',VOL.7,NO.5,MAY 1964. THE PROGRAM DOCUMEN- *
                                   11 *           TATION WILL FOLLOW THE NOTATION USED BY MARSAGLIA.   *
                                   12 ****************************************************************
                                   13          ENTRY RANEXP       PROLOGUE, STANDARD FOR 360/65.
000000                             14          USING RANEXP,15
000000 07                          15          DC    XL1'07'
000001 4009C1D5C5E7D7              16          DC    CL7' RANEXP'
000008 9019 D018           00018   17 RANEXP   STM   1,9,24(13)
00000C 47F0 F0AE           000B6   18 BRANCH   B     TEMP          EXECUTED ONLY ON THE FIRST CALL.
000010 3B00                        19          SER   0,0           RESULT SUMMED IN FPR0.
000012 5C40 F130           00138   20          M     4,BPAT        OBTAIN NEW U.
000016 5050 F128           00130   21          ST    5,LA          STORE FOR NEXT GENERATION.
00001A 1B66                        22          SR    6,6
00001C 1B44                        23 N1       SR    4,4           CLEAR R4.
00001E 8D40 0006           00006   24          SLDL  4,6           TWO HIGH ORDER OCTAL DIGITS OF U IN R4(D1D2)
000022 5940 F4B8           004C0   25          C     4,=F'37'      D1D2 COMPARED WITH 37(BASE 10)=45(BASE 8).
000026 47B0 F02E           00036   26          BC    11,NA         D1D2>=37 BRANCH NA.
00002A 4364 F492           0049A   27          IC    6,D+294(4)    OBTAIN DISCRETE RANDOM VARIABLE FROM D.
00002E 8950 0006           00006   28          SLL   5,6           R5=D5D6D7...
000032 47F0 F05A           00062   29          BC    15,EX         BRANCH EX.
000036 8D40 0003           00003   30 NA       SLDL  4,3           R4=D1D2D3.
00003A 5940 F4BC           004C4   31          C     4,=F'456'     456(BASE 10)=710(BASE 8).
00003E 47B0 F046           0004E   32          BC    11,NB         D1D2D3>=456 BRANCH NB.
000042 4364 F2EC           002F4   33          IC    6,D-144(4)    OBTAIN DISCRETE RANDOM VARIABLE FROM D.
000046 8950 0003           00003   34          SLL   5,3           R5=D5D6D7...
00004A 47F0 F05A           00062   35          BC    15,EX         BRANCH EX.
00004E 8D40 0003           00003   36 NB       SLDL  4,3           R4=D1D2D3D4.
000052 5940 F4C0           004C8   37          C     4,=F'3863'    3863(BASE 10)=7427(BASE 8).
000056 47B0 F072           0007A   38          BC    11,NC         D1D2D3D4>=3863 BRANCH NC.
00005A 5B40 F4C4           004CC   39          S     4,=F'3639'    OBTAIN PROPER ADDRESS FOR D TABLE.
00005E 4364 F36C           00374   40          IC    6,D(4)        OBTAIN DISCRETE VARIABLE FROM D.
000062 1846                        41 EX       LR    4,6           R4= DISCRETE VARIABLE.
000064 8C40 0010           00010   42 FL       SRDL  4,16          R5= DISCRETE VARIABLE + HI ORDER 16 BITS OF
000068 5050 F124           0012C   43          ST    5,RES         R5(NOW LO ORDER 16 BITS) WITH HI ORDER 8
00006C 9241 F124     0012C         44          MVI   RES,X'41'     BITS ZEROED OUT FOR CONVERSION TO FLOAT-
000070 7A00 F124           0012C   45          AE    0,RES         ING POINT. FPR0 CONTAINS RESULT + 'W'.
000074 9819 D018           00018   46          LM    1,9,24(13)      RESTORE REGISTERS.
000078 07FE                        47          BCR   15,14       BRANCH TO MAIN LINE.
00007A 5850 F128           00130   48 NC       L     5,LA          RESTORE U TO R5.
00007E 4120 0004           00004   49          LA    2,4
000082 1B33                        50          SR    3,3
000084 5553 F354           0035C   51          CL    5,P(3)        U>=P(3) BRANCH NO.
000088 47B0 F0FE           00106   52          BC    11,NO
00008C 1B32                        53          SR    3,2
00008E 1A32                        54 NE       AR    3,2           R3=IB(IN WORDS, 4 BYTES=1 WORD).
000090 5553 F154           0015C   55          CL    5,B(3)        U>=B(IB) BRANCH NE(CONTINUE LOOP).
```

```
LOC    OBJECT CODE    ADDR1 ADDR2  STMT   SOURCE STATEMENT                                          FU8APR70   6/30/71
000094 4780 F0A6            0008E    56            BC    11,NF
000098 5553 F234            0025C    57            CL    5,C(3)         C(IB)<U<B(IB) BRANCH NF.
00009C 4780 F0C8            000D0    58            BC    11,NF
0000A0 5C40 F130            00138    59            M     4,BPAT         OBTAIN NEW U.
0000A4 5050 F128            00130    60            ST    5,LA           STORE FOR NEXT GENERATIO.
0000A8 1B44                          61 GO         SR    4,4            CLEAR R4.
0000AA 8830 0002            00002    62            SRL   3,2            CONVERT IR TO BYTES.
0000AE 4343 F36C            00374    63            IC    4,D(3)         OBTAIN DISCRETE VARIABLE.
0000B2 47F0 F05C            00064    64            BC    15,FL          BRANCH TO FL.
0000B6 5850 F128            00130    65 TEMP       L     5,LA           REPLACES INSTR. AT BRANCH.
0000BA D203 F004 F0AE 0000C 000B6    66            MVC   BRANCH,TEMP    REPLACE BRANCH BY TEMP.
0000C0 5850 1000            00000    67            L     5,0(,1)        ADDRESS OF PRIMER IN R5.
0000C4 5850 5000            00000    68            L     5,0(,5)        PRIMER IN R5.
0000C8 5050 F128            00130    69            ST    5,LA           STORE PRIMER FOR USE IN URAN.
0000CC 47F0 F008            00010    70            B     BRANCH+4,      BRANCH TO INSTRUCTION FOLLOWING 'BRANCH'.
0000D0 5C40 F130            00138    71 NF         M     4,BPAT         OBTAIN NEW U.
0000D4 1882                          72            LR    8,2            USED FOR INDEXING.
0000D6 1A82                          73            AR    8,2            R8=IQ.
0000D8 5558 F134            0013C    74            CL    5,Q(8)         U>=Q(IQ) CONTINUE LOOP.
0000DC 4780 F0CE            000D6    75            BC    11,*-6
0000E0 5C40 F130            00138    76            M     4,BPAT         OBTAIN NEW U.
0000E4 1B82                          77            SR    8,2
0000E6 1865                          78            LR    6,5
0000E8 1192                          79            LNR   9,2            R9=-4.
0000EA 1B82                          80            SR    8,2
0000EC 5C40 F130            00138    81 NH         M     4,BPAT         OBTAIN NEW U.
0000F0 1556                          82            CLR   5,6            R5>=R6 BRANCH TO CONTINUE.
0000F2 4780 F0F0            000F8    83            BC    11,*+6
0000F6 1865                          84            LR    6,5            LOAD SMALLEST U IN R6.
0000F8 8689 F0E4            000EC    85            BXH   8,9,NH         CONTINUE(R9 HAS BOTH INCREMENT & DELIMITER).
0000FC 5050 F128            00130    86            ST    5,LA
000100 1856                          87            LR    5,6            R5=MIN(U1,U2,...,U(IQ)).
000102 47F0 F0A0            000A8    88            BC    15,GO
000106 1A32                          89 ND         AR    3,2            R3=IP.
000108 5553 F354            0035C    90            CL    5,P(3)         U>=P(IP) CONTINUE LOOP.
00010C 4780 F0FE            00106    91            BC    11,ND
000110 5030 F12C            00134    92            ST    3,J            STORE IP INTO J.
000114 9246 F12C      00134          93            MVI   J,X'46'        CONVERT IP TO FLOATING POINT.
000118 7A00 F12C            00134    94            AE    0,J            NORMALIZED AND LOADED INTO FPR0 'W'.
00011C 5C40 F130            00138    95            M     4,BPAT         OBTAIN NEW U.
000120 5050 F128            00130    96            ST    5,LA           STORE FOR NEXT GENERATION.
000124 5C45 F354            0035C    97            M     4,P(6)         U=U*P(Q).
000128 47F0 F014            0001C    98            BC    15,NI          BRANCH NI.
00012C                               99 RES        DS    1F
000130                              100 LA         DS    1F
000134                              101 J          DS    1F
000138 48C27395                     102 BPAT       DC    XL4'48C27395'
00013C 00000000                     103 Q          DC    X'00000000'     BASE 10 0.0
000140 00000000                     104            DC    X'00000000'     BASE 10 0.0
000144 FA81CEA7                     105            DC    X'FA81CEA7'     BASE 10= 0.97927562333
000148 FFEAD84B                     106            DC    X'FFEAD84B'     BASE 10= 0.99967719882
00014C FFFFBC72                     107            DC    X'FFFFBC72'     BASE 10= 0.99999597344
000150 FFFFFF4C                     108            DC    X'FFFFFF4C'     BASE 10= 0.99999995812
000154 FFFFFFFE                     109            DC    X'FFFFFFFE'     BASE 10= 0.99999999963
000158 FFFFFFFF                     110            DC    X'FFFFFFFF'     BASE 10= 1.00000000000
```

```
  LCC  OBJECT CODE     ADDR1 ADDR2  STMT    SOURCE STATEMENT

00015C F1F2A021                     111 B       DC      X'F1F2A021'     BASE 10 0.94510842156
000160 F23FF28D                     112         DC      X'F23FF28D'     BASE 10 0.94678827211
000164 F28C810C                     113         DC      X'F28C810C'     BASE 10 0.94745642234
000168 F2C13730                     114         DC      X'F2C13730'     BASE 10 0.94826073593
00016C F2FDF518                     115         DC      X'F2EDE518'     BASE 10 0.94896248988
000170 F307428B                     116         DC      X'F307428B'     BASE 10 0.94932953914
000174 F317F405                     117         DC      X'F317F405'     BASE 10 0.94958129319
000178 F326185C                     118         DC      X'F326185C'     BASE 10 0.94989903601
00017C F330F665                     119         DC      X'F330F665'     BASE 10 0.94992008199
000180 F3B0051C                     120         DC      X'F3B0051C'     BASE 10 0.95190460160
000184 F42015AE                     121         DC      X'F42015AE'     BASE 10 0.95361457350
000188 F4B0094E                     122         DC      X'F4B0094F'     BASE 10 0.95525892423
00018C F4F4020F                     123         DC      X'F4F4020F'     BASE 10 0.95606066501
000190 F5592594                     124         DC      X'F5592574'     BASE 10 0.95839152226
000194 F592107A                     125         DC      X'F592107A'     BASE 10 0.95974907150
000198 F60A64D7                     126         DC      X'F60A64D7'     BASE 10 0.96109651596
00019C F65F0C8F                     127         DC      X'F65F0C8F'     BASE 10 0.96237258570
0001A0 F6A471A9                     128         DC      X'F6A471A9'     BASE 10 0.96344671629
0001A4 F6E809E1                     129         DC      X'F6E809E1'     BASE 10 0.96447806832
0001A8 F729F034                     130         DC      X'F729F034'     BASE 10 0.96549445247
0001AC F761312A                     131         DC      X'F761312A'     BASE 10 0.96632578298
0001B0 F7941C80                     132         DC      X'F7941C80'     BASE 10 0.96710374973
0001B4 F7C596FF                     133         DC      X'F7C586EF'     BASE 10 0.96785063536
0001B8 F7F1E012                     134         DC      X'F7F1E012'     BASE 10 0.96853524871
0001BC F81DD47F                     135         DC      X'F81DD47F'     BASE 10 0.96920558791
0001C0 F849D468                     136         DC      X'F849D468'     BASE 10 0.96986129414
0001C4 F871E734                     137         DC      X'F871E734'     BASE 10 0.97048802404
0001C8 F897DE94                     138         DC      X'F897DE94'     BASE 10 0.97106734391
0001CC F8B9DC57                     139         DC      X'F8B9DC57'     BASE 10 0.97159600928
0001D0 F8DAC69F                     140         DC      X'F8DAC69F'     BASE 10 0.97208825453
0001D4 F8F9106D                     141         DC      X'F8F9106D'     BASE 10 0.97254093572
0001D8 F91A299F                     142         DC      X'F91A299F'     BASE 10 0.97305557854
0001DC F938A989                     143         DC      X'F938A989'     BASE 10 0.97352084759
0001E0 F956965B                     144         DC      X'F956965B'     BASE 10 0.97397651342
0001E4 F9749441                     145         DC      X'F9749441'     BASE 10 0.97442652313
0001E8 F9900A1A                     146         DC      X'F9900A1A'     BASE 10 0.97485411775
0001EC F9AA4357                     147         DC      X'F9AA4357'     BASE 10 0.97525425799
0001F0 F9C36980                     148         DC      X'F9C36980'     BASE 10 0.97563794277
0001F4 F9DC3773                     149         DC      X'F9DC3773'     BASE 10 0.97601648869
0001F8 F9F46745                     150         DC      X'F9F46745'     BASE 10 0.97638554992
0001FC FA0A55CA                     151         DC      X'FA0A559A'     BASE 10 0.97672015570
000200 FA20C49B                     152         DC      X'FA20C49B'     BASE 10 0.97705105581
000204 FA33E7C0                     153         DC      X'FA33E7C0'     BASE 10 0.97735451174
000208 FA4657F8                     154         DC      X'FA4657F8'     BASE 10 0.97763681231
00020C FA580771                     155         DC      X'FA580771'     BASE 10 0.97791811485
000210 FA6B3FD0                     156         DC      X'FA6B3FD0'     BASE 10 0.97819699411
000214 FA7D6E58                     157         DC      X'FA7D6E58'     BASE 10 0.97847642589
000218 FA8EC1BA                     158         DC      X'FA8EC19A'     BASE 10 0.97874079506
00021C FA9F650C                     159         DC      X'FA9F650C'     BASE 10 0.97899467042
000220 FAAF65EE                     160         DC      X'FAAF65EE'     BASE 10 0.97923892340
000224 FABF09A2                     161         DC      X'FABF09A2'     BASE 10 0.97947750293
000228 FACEA835                     162         DC      X'FACEA935'     BASE 10 0.97971601531
00022C FADD203A                     163         DC      X'FADD203A'     BASE 10 0.97993661330
000230 FAEB949A                     164         DC      X'FAEB949A'     BASE 10 0.98015717294
000234 FAF96A01                     165         DC      X'FAF96A01'     BASE 10 0.98036825688
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  LOC  OBJECT CODE    ADDR1 ADDR2  STMT   SOURCE STATEMENT

000238 FB0AD436                     166         DC    X'FB0AD436'    BASE 10 0.98057295140
00023C FB14358A                     167         DC    X'FB14358A'    BASE 10 0.98077711667
000240 FB1F2894                     168         DC    X'FB1F2894'    BASE 10 0.98094419111
000244 FB2910CA                     169         DC    X'FB2910CA'    BASE 10 0.98109613598
000248 FB32EF11                     170         DC    X'FB32EF11'    BASE 10 0.98124587942
00024C FB3CA379                     171         DC    X'FB3CA379'    BASE 10 0.98139402130
000250 FB44B4B4                     172         DC    X'FB44B4B4'    BASE 10 0.98151460399
000254 FB4C437C                     173         DC    X'FB4C437C'    BASE 10 0.98163244057
000258 FB4FAA91                     174         DC    X'FB4FAA91'    BASE 10 0.98169436111
00025C F17705FD                     175 C       DC    X'F17705FD'    BASE 10 0.94323490080
000260 F1FF3941                     176         DC    X'F1FF3941'    BASE 10 0.94529539518
000264 F24EC2F8                     177         DC    X'F24EC2F8'    BASE 10 0.94651430666
000268 F2943837                     178         DC    X'F2943837'    BASE 10 0.94757146938
00026C F2CCD8B9                     179         DC    X'F2CCD8B9'    BASE 10 0.94843821067
000270 F2F6A495                     180         DC    X'F2F6A495'    BASE 10 0.94907597217
000274 F3136908                     181         DC    X'F3136908'    BASE 10 0.94951504656
000278 F31D803D                     182         DC    X'F31D803D'    BASE 10 0.94947253451
00027C F329C2C7                     183         DC    X'F329C2C7'    BASE 10 0.94985597021
000280 F33CA878                     184         DC    X'F33CA878'    BASE 10 0.95014449778
000284 F3838926                     185         DC    X'F3838926'    BASE 10 0.95196110898
000288 F426007D                     186         DC    X'F426007D'    BASE 10 0.95373563807
00028C F49AFC59                     187         DC    X'F49AFC59'    BASE 10 0.95549897478
000290 F4E936A6                     188         DC    X'F4E936A6'    BASE 10 0.95693234495
000294 F5509921                     189         DC    X'F5509921'    BASE 10 0.95846135204
000298 F5AEF212                     190         DC    X'F5AEF212'    BASE 10 0.95996016747
00029C F60EC556                     191         DC    X'F60EC556'    BASE 10 0.96116289522
0002A0 F65E7B07                     192         DC    X'F65E7B07'    BASE 10 0.96237915936
0002A4 F6AEC971                     193         DC    X'F6AEC971'    BASE 10 0.96359303254
0002A8 F6F39363                     194         DC    X'F6F39363'    BASE 10 0.96465303933
0002AC F72E0177                     195         DC    X'F72E0177'    BASE 10 0.96554574164
0002B0 F7640AB7                     196         DC    X'F7640AB7'    BASE 10 0.96637302930
0002B4 F7A23BE7                     197         DC    X'F7A23BE7'    BASE 10 0.96732393403
0002B8 F7C77089                     198         DC    X'F7C77089'    BASE 10 0.96788774270
0002BC F7FFCFB3                     199         DC    X'F7FFCFB3'    BASE 10 0.96873186249
0002C0 F823515C                     200         DC    X'F823515C'    BASE 10 0.96928987209
0002C4 F84AD4DB                     201         DC    X'F84AD4DB'    BASE 10 0.96987974944
0002C8 F87C79AC                     202         DC    X'F87C79AC'    BASE 10 0.97064928261
0002CC F8A455F7                     203         DC    X'F8A455F7'    BASE 10 0.97126042281
0002D0 F8C7E209                     204         DC    X'F8C7E209'    BASE 10 0.97180092548
0002D4 F8E15379                     205         DC    X'F8E15379'    BASE 10 0.97219820298
0002D8 F8FD015B                     206         DC    X'F8FD015B'    BASE 10 0.97261055461
0002DC F921F2CE                     207         DC    X'F921F2CE'    BASE 10 0.97317426251
0002E0 F93F5398                     208         DC    X'F93F5398'    BASE 10 0.97360757567
0002E4 F9663071                     209         DC    X'F9663071'    BASE 10 0.97421630871
0002E8 F97E5995                     210         DC    X'F97E5995'    BASE 10 0.97458419708
0002EC F9909911                     211         DC    X'F9909911'    BASE 10 0.97503048590
0002F0 F9B7401A                     212         DC    X'F9B7401A'    BASE 10 0.97545242919
0002F4 F9CC980C                     213         DC    X'F9CC980C'    BASE 10 0.97577828456
0002F8 F9E759C9                     214         DC    X'F9E759C9'    BASE 10 0.97607968974
0002FC FA094CC4                     215         DC    X'FA094CC4'    BASE 10 0.97656636054
000300 FA19E797                     216         DC    X'FA19E797'    BASE 10 0.97695777372
000304 FA2ED64F                     217         DC    X'FA2ED64F'    BASE 10 0.97727717817
000308 FA337605                     218         DC    X'FA337605'    BASE 10 0.97743933414
00030C FA52557E                     219         DC    X'FA52557E'    BASE 10 0.97781981658
000310 FA66B368                     220         DC    X'FA66B368'    BASE 10 0.97816010835
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LOC	OBJECT CODE	ADDR1	ADDR2	STMT	SOURCE STATEMENT		
000314	FA73F667			221	DC	X'FA73F667'	BASE 10 0.97831194768
000318	FA8355F5			222	DC	X'FA8355F5'	BASE 10 0.97856652118
00031C	FA94E954			223	DC	X'FA94E954'	BASE 10 0.97894151990
000320	FAAB798A			224	DC	X'FAAB798A'	BASE 10 0.97913322095
000324	FAB8172F			225	DC	X'FAB8172F'	BASE 10 0.97941727549
000328	FAC5C67C			226	DC	X'FAC5C67C'	BASE 10 0.97959020048
00032C	FAD7861A			227	DC	X'FAD7861A'	BASE 10 0.97985429215
000330	FAE434D6			228	DC	X'FAE434D6'	BASE 10 0.98004465[illegible]
000334	FAF1907C			229	DC	X'FAF1907C'	BASE 10 0.98024840031
000338	FB044F58			230	DC	X'FB044F58'	BASE 10 0.98053164218
00033C	FB115246			231	DC	X'FB115246'	BASE 10 0.98073305333
000340	FB146DE4			232	DC	X'FB146DE4'	BASE 10 0.98078047595
000344	FB245AF4			233	DC	X'FB245AF4'	BASE 10 0.98102344782
000348	FB2FA86D			234	DC	X'FB2FA86D'	BASE 10 0.98119594912
00034C	FB36E54B			235	DC	X'FB36E54B'	BASE 10 0.98130639389
000350	FB40D549			236	DC	X'FB40D549'	BASE 10 0.98145802546
000354	FB49BD34			237	DC	X'FB49BD34'	BASE 10 0.98159104669
000358	FB4C97A1			238	DC	X'FB4C97A1'	BASE 10 0.98163745593
00035C	FB4FAA91			239 P	DC	X'FB4FAA91'	BASE 10 0.98168436111
000360	FFFA03DE			240	DC	X'FFFA03DE'	BASE 10 0.99966653737
000364	FFFF9BEA			241	DC	X'FFFF9BEA'	BASE 10 0.99999385579
000368	FFFFFE1C			242	DC	X'FFFFFE1C'	BASE 10 0.99999988746
00036C	FFFFFFF7			243	DC	X'FFFFFFF7'	BASE 10 0.99999999794
000370	FFFFFFFF			244	DC	X'FFFFFFFF'	BASE 10 0.99999999996
000374	00			245 D	DC	X'00'	BASE 10 0.0
000375	0A			246	DC	X'0A'	BASE 10 0.6250
000376	0B			247	DC	X'0B'	BASE 10 0.6875
000377	10			248	DC	X'10'	BASE 10 1.0000
000378	15			249	DC	X'15'	BASE 10 1.3125
000379	20			250	DC	X'20'	BASE 10 2.0000
00037A	35			251	DC	X'35'	BASE 10 3.3125
00037B	2B			252	DC	X'2B'	BASE 10 2.6875
00037C	36			253	DC	X'36'	BASE 10 3.3750
00037D	01			254	DC	X'01'	BASE 10 0.0625
00037E	02			255	DC	X'02'	BASE 10 0.1250
00037F	03			256	DC	X'03'	BASE 10 0.1875
000380	05			257	DC	X'05'	BASE 10 0.3125
000381	04			258	DC	X'04'	BASE 10 0.2500
000382	06			259	DC	X'06'	BASE 10 0.3750
000383	08			260	DC	X'08'	BASE 10 0.5000
000384	07			261	DC	X'07'	BASE 10 0.4375
000385	09			262	DC	X'09'	BASE 10 0.5625
000386	0C			263	DC	X'0C'	BASE 10 0.7500
000387	0D			264*	DC	X'0D'	BASE 10 0.8125
000388	0E			265	DC	X'0E'	BASE 10 0.8750
000389	0F			266	DC	X'0F'	BASE 10 0.9375
00038A	14			267	DC	X'14'	BASE 10 1.2500
00038B	11			268	DC	X'11'	BASE 10 1.0625
00038C	16			269	DC	X'16'	BASE 10 1.3750
00038D	13			270	DC	X'13'	BASE 10 1.1875
00038E	12			271	DC	X'12'	BASE 10 1.1250
00038F	18			272	DC	X'18'	BASE 10 1.5000
000390	1C			273	DC	X'1C'	BASE 10 1.7500
000391	1E			274	DC	X'1E'	BASE 10 1.8750
000392	19			275	DC	X'19'	BASE 10 1.5625

LOC	OBJECT CODE	ADDR1	ADDR2	STMT	SOURCE STATEMENT		
000393	17			276	DC	X'17'	BASE 10 1.4375
000394	1B			277	DC	X'1B'	BASE 10 1.6875
000395	1A			278	DC	X'1A'	BASE 10 1.6250
000396	23			279	DC	X'23'	BASE 10 2.1875
000397	1F			280	DC	X'1F'	BASE 10 1.9375
000398	22			281	DC	X'22'	BASE 10 2.1250
000399	25			282	DC	X'25'	BASE 10 2.3125
00039A	21			283	DC	X'21'	BASE 10 2.0625
00039B	1D			284	DC	X'1D'	BASE 10 1.8125
00039C	28			285	DC	X'28'	BASE 10 2.5000
00039D	30			286	DC	X'30'	BASE 10 3.0000
00039E	33			287	DC	X'33'	BASE 10 3.1875
00039F	24			288	DC	X'24'	BASE 10 2.2500
0003A0	2F			289	DC	X'2F'	BASE 10 2.9375
0003A1	3E			290	DC	X'3E'	BASE 10 3.8750
0003A2	29			291	DC	X'29'	BASE 10 2.5625
0003A3	26			292	DC	X'26'	BASE 10 2.3750
0003A4	39			293	DC	X'39'	BASE 10 3.5625
0003A5	2E			294	DC	X'2E'	BASE 10 2.8750
0003A6	37			295	DC	X'37'	BASE 10 3.4375
0003A7	2A			296	DC	X'2A'	BASE 10 2.6250
0003A8	32			297	DC	X'32'	BASE 10 3.1250
0003A9	2D			298	DC	X'2D'	BASE 10 2.8125
0003AA	2C			299	DC	X'2C'	BASE 10 2.7500
0003AB	3F			300	DC	X'3F'	BASE 10 3.9375
0003AC	3C			301	DC	X'3C'	BASE 10 3.7500
0003AD	27			302	DC	X'27'	BASE 10 2.4375
0003AE	34			303	DC	X'34'	BASE 10 3.2500
0003AF	3A			304	DC	X'3A'	BASE 10 3.6250
0003B0	31			305	DC	X'31'	BASE 10 3.0625
0003B1	38			306	DC	X'38'	BASE 10 3.5000
0003B2	3D			307	DC	X'3D'	BASE 10 3.8125
0003B3	3B			308	DC	X'3B'	BASE 10 3.6875
0003B4	02			309	DC	X'02'	BASE 10 0.1250
0003B5	03			310	DC	X'03'	BASE 10 0.1875
0003B6	03			311	DC	X'03'	BASE 10 0.1875
0003B7	03			312	DC	X'03'	BASE 10 0.1875
0003B8	03			313	DC	X'03'	BASE 10 0.1875
0003B9	03			314	DC	X'03'	BASE 10 0.1875
0003BA	03			315	DC	X'03'	BASE 10 0.1875
0003BB	05			316	DC	X'05'	BASE 10 0.3125
0003BC	0F			317	DC	X'0F'	BASE 10 0.9375
0003BD	0F			318	DC	X'0F'	BASE 10 0.9375
0003BE	14			319	DC	X'14'	BASE 10 1.2500
0003BF	14			320	DC	X'14'	BASE 10 1.2500
0003C0	14			321	DC	X'14'	BASE 10 1.2500
0003C1	12			322	DC	X'12'	BASE 10 1.1250
0003C2	12			323	DC	X'12'	BASE 10 1.1250
0003C3	12			324	DC	X'12'	BASE 10 1.1250
0003C4	12			325	DC	X'12'	BASE 10 1.1250
0003C5	1A			326	DC	X'1A'	BASE 10 1.6250
0003C6	25			327	DC	X'25'	BASE 10 2.3125
0003C7	25			328	DC	X'25'	BASE 10 2.3125
0003C8	25			329	DC	X'25'	BASE 10 2.3125
0003C9	25			330	DC	X'25'	BASE 10 2.3125

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  LOC  OBJECT CODE    ADDR1 ADDR2  STMT   SOURCE STATEMENT

0003CA 21                           331          DC     X'21'          BASE 10  2.0625
0003CB 21                           332          DC     X'21'          BASE 10  2.0625
0003CC 1D                           333          DC     X'1D'          BASE 10  1.8125
0003CD 1D                           334          DC     X'1D'          BASE 10  1.8125
0003CE 30                           335          DC     X'30'          BASE 10  3.0000
0003CF 2F                           336          DC     X'2F'          BASE 10  2.9375
0003D0 2F                           337          DC     X'2F'          BASE 10  2.9375
0003D1 3E                           338          DC     X'3E'          BASE 10  3.8750
0003D2 3E                           339          DC     X'3E'          BASE 10  3.8750
0003D3 3E                           340          DC     X'3E'          BASE 10  3.8750
0003D4 26                           341          DC     X'26'          BASE 10  2.3750
0003D5 26                           342          DC     X'26'          BASE 10  2.3750
0003D6 26                           343          DC     X'26'          BASE 10  2.3750
0003D7 39                           344          DC     X'39'          BASE 10  3.5625
0003D8 39                           345          DC     X'39'          BASE 10  3.5625
0003D9 39                           346          DC     X'39'          BASE 10  3.5625
0003DA 39                           347          DC     X'39'          BASE 10  3.5625
0003DB 39                           348          DC     X'39'          BASE 10  3.5625
0003DC 2E                           349          DC     X'2E'          BASE 10  2.8750
0003DD 2E                           350          DC     X'2E'          BASE 10  2.8750
0003DE 2E                           351          DC     X'2E'          BASE 10  2.8750
0003DF 37                           352          DC     X'37'          BASE 10  3.4375
0003E0 37                           353          DC     X'37'          BASE 10  3.4375
0003E1 37                           354          DC     X'37'          BASE 10  3.4375
0003E2 37                           355          DC     X'37'          BASE 10  3.4375
0003E3 37                           356          DC     X'37'          BASE 10  3.4375
0003E4 37                           357          DC     X'37'          BASE 10  3.4375
0003E5 2D                           358          DC     X'2D'          BASE 10  2.8125
0003E6 2D                           359          DC     X'2D'          BASE 10  2.8125
0003E7 2D                           360          DC     X'2D'          BASE 10  2.8125
0003E8 2D                           361          DC     X'2D'          BASE 10  2.8125
0003E9 2C                           362          DC     X'2C'          BASE 10  2.7500
0003EA 2C                           363          DC     X'2C'          BASE 10  2.7500
0003EB 2C                           364          DC     X'2C'          BASE 10  2.7500
0003EC 2C                           365          DC     X'2C'          BASE 10  2.7500
0003ED 2C                           366          DC     X'2C'          BASE 10  2.7500
0003EE 3F                           367          DC     X'3F'          BASE 10  3.9375
0003EF 3F                           368          DC     X'3F'          BASE 10  3.9375
0003F0 3F                           369          DC     X'3F'          BASE 10  3.9375
0003F1 3C                           370          DC     X'3C'          BASE 10  3.7500
0003F2 3C                           371          DC     X'3C'          BASE 10  3.7500
0003F3 3C                           372          DC     X'3C'          BASE 10  3.7500
0003F4 3C                           373          DC     X'3C'          BASE 10  3.7500
0003F5 27                           374          DC     X'27'          BASE 10  2.4375
0003F6 27                           375          DC     X'27'          BASE 10  2.4375
0003F7 3A                           376          DC     X'3A'          BASE 10  3.6250
0003F8 3A                           377          DC     X'3A'          BASE 10  3.6250
0003F9 3A                           378          DC     X'3A'          BASE 10  3.6250
0003FA 3A                           379          DC     X'3A'          BASE 10  3.6250
0003FB 3A                           380          DC     X'3A'          BASE 10  3.6250
0003FC 31                           381          DC     X'31'          BASE 10  3.0625
0003FD 38                           382          DC     X'38'          BASE 10  3.5000
0003FE 38                           383          DC     X'38'          BASE 10  3.5000
0003FF 38                           384          DC     X'38'          BASE 10  3.5000
000400 38                           385          DC     X'38'          BASE 10  3.5000
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  LOC  OBJECT CODE    ADDR1 ADDR2  STMT   SOURCE STATEMENT

000401 38                          386      DC    X'38'        BASE 10  3.5000
000402 38                          387      DC    X'38'        BASE 10  3.5000
000403 3D                          388      DC    X'3D'        BASE 10  3.8125
000404 3D                          389      DC    X'3D'        BASE 10  3.8125
000405 3D                          390      DC    X'3D'        BASE 10  3.8125
000406 3D                          391      DC    X'3D'        BASE 10  3.8125
000407 3B                          392      DC    X'3B'        BASE 10  3.6875
000408 3B                          393      DC    X'3B'        BASE 10  3.6875
000409 3B                          394      DC    X'3B'        BASE 10  3.6875
00040A 3B                          395      DC    X'3B'        BASE 10  3.6875
00040B 3B                          396      DC    X'3B'        BASE 10  3.6875
00040C 02                          397      DC    X'02'        BASE 10  0.1250
00040D 02                          398      DC    X'02'        BASE 10  0.1250
00040E 05                          399      DC    X'05'        BASE 10  0.3125
00040F 05                          400      DC    X'05'        BASE 10  0.3125
000410 05                          401      DC    X'05'        BASE 10  0.3125
000411 05                          402      DC    X'05'        BASE 10  0.3125
000412 05                          403      DC    X'05'        BASE 10  0.3125
000413 04                          404      DC    X'04'        BASE 10  0.2500
000414 04                          405      DC    X'04'        BASE 10  0.2500
000415 06                          406      DC    X'06'        BASE 10  0.3750
000416 06                          407      DC    X'06'        BASE 10  0.3750
000417 06                          408      DC    X'06'        BASE 10  0.3750
000418 06                          409      DC    X'06'        BASE 10  0.3750
000419 07                          410      DC    X'07'        BASE 10  0.4375
00041A 07                          411      DC    X'07'        BASE 10  0.4375
00041B 0D                          412      DC    X'0D'        BASE 10  0.8125
00041C 0E                          413      DC    X'0E'        BASE 10  0.8750
00041D 0E                          414      DC    X'0E'        BASE 10  0.8750
00041E 0E                          415      DC    X'0E'        BASE 10  0.8750
00041F 0F                          416      DC    X'0F'        BASE 10  0.9375
000420 0F                          417      DC    X'0F'        BASE 10  0.9375
000421 0F                          418      DC    X'0F'        BASE 10  0.9375
000422 11                          419      DC    X'11'        BASE 10  1.0625
000423 11                          420      DC    X'11'        BASE 10  1.0625
000424 16                          421      DC    X'16'        BASE 10  1.3750
000425 16                          422      DC    X'16'        BASE 10  1.3750
000426 16                          423      DC    X'16'        BASE 10  1.3750
000427 12                          424      DC    X'12'        BASE 10  1.1250
000428 18                          425      DC    X'18'        BASE 10  1.5000
000429 18                          426      DC    X'18'        BASE 10  1.5000
00042A 18                          427      DC    X'18'        BASE 10  1.5000
00042B 18                          428      DC    X'18'        BASE 10  1.5000
00042C 1E                          429      DC    X'1E'        BASE 10  1.8750
00042D 1E                          430      DC    X'1E'        BASE 10  1.8750
00042E 1E                          431      DC    X'1E'        BASE 10  1.8750
00042F 19                          432      DC    X'19'        BASE 10  1.5625
000430 1B                          433      DC    X'1B'        BASE 10  1.6875
000431 1B                          434      DC    X'1B'        BASE 10  1.6875
000432 1B                          435      DC    X'1B'        BASE 10  1.6875
000433 1A                          436      DC    X'1A'        BASE 10  1.6250
000434 1A                          437      DC    X'1A'        BASE 10  1.6250
000435 1A                          438      DC    X'1A'        BASE 10  1.6250
000436 1A                          439      DC    X'1A'        BASE 10  1.6250
000437 1A                          440      DC    X'1A'        BASE 10  1.6250
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  LOC  OBJECT CODE    ADDR1 ADDR2  STMT   SOURCE STATEMENT
000438 23                           441         DC     X'23'          BASE 10  2.1875
000439 1F                           442         DC     X'1F'          BASE 10  1.9375
00043A 22                           443         DC     X'22'          BASE 10  2.1250
00043B 22                           444         DC     X'22'          BASE 10  2.1250
00043C 22                           445         DC     X'22'          BASE 10  2.1250
00043D 25                           446         DC     X'25'          BASE 10  2.3125
00043E 25                           447         DC     X'25'          BASE 10  2.3125
00043F 21                           448         DC     X'21'          BASE 10  2.0625
000440 21                           449         DC     X'21'          BASE 10  2.0625
000441 21                           450         DC     X'21'          BASE 10  2.0625
000442 1D                           451         DC     X'1D'          BASE 10  1.8125
000443 1D                           452         DC     X'1D'          BASE 10  1.8125
000444 1D                           453         DC     X'1D'          BASE 10  1.8125
000445 1D                           454         DC     X'1D'          BASE 10  1.8125
000446 28                           455         DC     X'28'          BASE 10  2.5000
000447 28                           456         DC     X'28'          BASE 10  2.5000
000448 30                           457         DC     X'30'          BASE 10  3.0000
000449 2F                           458         DC     X'2F'          BASE 10  2.9375
00044A 29                           459         DC     X'29'          BASE 10  2.5625
00044B 26                           460         DC     X'26'          BASE 10  2.3750
00044C 26                           461         DC     X'26'          BASE 10  2.3750
00044D 2E                           462         DC     X'2E'          BASE 10  2.8750
00044E 32                           463         DC     X'32'          BASE 10  3.1250
00044F 2D                           464         DC     X'2D'          BASE 10  2.8125
000450 2C                           465         DC     X'2C'          BASE 10  2.7500
000451 27                           466         DC     X'27'          BASE 10  2.4375
000452 27                           467         DC     X'27'          BASE 10  2.4375
000453 31                           468         DC     X'31'          BASE 10  3.0625
000454 00                           469         DC     X'00'          BASE 10  0.0000
000455 00                           470         DC     X'00'          BASE 10  0.0000
000456 00                           471         DC     X'00'          BASE 10  0.0000
000457 01                           472         DC     X'01'          BASE 10  0.0625
000458 04                           473         DC     X'04'          BASE 10  0.2500
000459 04                           474         DC     X'04'          BASE 10  0.2500
00045A 04                           475         DC     X'04'          BASE 10  0.2500
00045B 0B                           476         DC     X'0B'          BASE 10  0.6875
00045C 0B                           477         DC     X'0B'          BASE 10  0.6875
00045D 0B                           478         DC     X'0B'          BASE 10  0.6875
00045E 0B                           479         DC     X'0B'          BASE 10  0.6875
00045F 0B                           480         DC     X'0B'          BASE 10  0.6875
000460 0B                           481         DC     X'0B'          BASE 10  0.6875
000461 0C                           482         DC     X'0C'          BASE 10  0.7500
000462 0C                           483         DC     X'0C'          BASE 10  0.7500
000463 0C                           484         DC     X'0C'          BASE 10  0.7500
000464 0C                           485         DC     X'0C'          BASE 10  0.7500
000465 0C                           486         DC     X'0C'          BASE 10  0.7500
000466 0D                           487         DC     X'0D'          BASE 10  0.8125
000467 0D                           488         DC     X'0D'          BASE 10  0.8125
000468 0D                           489         DC     X'0D'          BASE 10  0.8125
000469 10                           490         DC     X'10'          BASE 10  1.0000
00046A 10                           491         DC     X'10'          BASE 10  1.0000
00046B 16                           492         DC     X'16'          BASE 10  1.3750
00046C 16                           493         DC     X'16'          BASE 10  1.3750
00046D 16                           494         DC     X'16'          BASE 10  1.3750
00046E 16                           495         DC     X'16'          BASE 10  1.3750
```

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 LOC  OBJECT CODE    ADDR1 ADDR2  STMT   SOURCE STATEMENT

00046F 18                          496          DC    X'18'          BASE 10  1.5000
000470 18                          497          DC    X'18'          BASE 10  1.5000
000471 1C                          498          DC    X'1C'          BASE 10  1.7500
000472 1C                          499          DC    X'1C'          BASE 10  1.7500
000473 1C                          500          DC    X'1C'          BASE 10  1.7500
000474 1C                          501          DC    X'1C'          BASE 10  1.7500
000475 1C                          502          DC    X'1C'          BASE 10  1.7500
000476 1E                          503          DC    X'1E'          BASE 10  1.8750
000477 19                          504          DC    X'19'          BASE 10  1.5625
000478 19                          505          DC    X'19'          BASE 10  1.5625
000479 19                          506          DC    X'19'          BASE 10  1.5625
00047A 19                          507          DC    X'19'          BASE 10  1.5625
00047B 19                          508          DC    X'19'          BASE 10  1.5625
00047C 17                          509          DC    X'17'          BASE 10  1.4375
00047D 17                          510          DC    X'17'          BASE 10  1.4375
00047E 17                          511          DC    X'17'          BASE 10  1.4375
00047F 17                          512          DC    X'17'          BASE 10  1.4375
000480 17                          513          DC    X'17'          BASE 10  1.4375
000481 17                          514          DC    X'17'          BASE 10  1.4375
000482 17                          515          DC    X'17'          BASE 10  1.4375
000483 1B                          516          DC    X'1B'          BASE 10  1.6875
000484 1B                          517          DC    X'1B'          BASE 10  1.6875
000485 23                          518          DC    X'23'          BASE 10  2.1875
000486 23                          519          DC    X'23'          BASE 10  2.1875
000487 1F                          520          DC    X'1F'          BASE 10  1.9375
000488 1F                          521          DC    X'1F'          BASE 10  1.9375
000489 1F                          522          DC    X'1F'          BASE 10  1.9375
00048A 20                          523          DC    X'20'          BASE 10  2.0000
00048B 20                          524          DC    X'20'          BASE 10  2.0000
00048C 20                          525          DC    X'20'          BASE 10  2.0000
00048D 20                          526          DC    X'20'          BASE 10  2.0000
00048E 33                          527          DC    X'33'          BASE 10  3.1875
00048F 24                          528          DC    X'24'          BASE 10  2.2500
000490 24                          529          DC    X'24'          BASE 10  2.2500
000491 24                          530          DC    X'24'          BASE 10  2.2500
000492 29                          531          DC    X'29'          BASE 10  2.5625
000493 35                          532          DC    X'35'          BASE 10  3.3125
000494 2A                          533          DC    X'2A'          BASE 10  2.6250
000495 2A                          534          DC    X'2A'          BASE 10  2.6250
000496 2B                          535          DC    X'2B'          BASE 10  2.6875
000497 2B                          536          DC    X'2B'          BASE 10  2.6875
000498 34                          537          DC    X'34'          BASE 10  3.2500
000499 36                          538          DC    X'36'          BASE 10  3.3750
00049A 00                          539          DC    X'00'          BASE 10  0.0000
00049B 00                          540          DC    X'00'          BASE 10  0.0000
00049C 00                          541          DC    X'00'          BASE 10  0.0000
00049D 01                          542          DC    X'01'          BASE 10  0.0625
00049E 01                          543          DC    X'01'          BASE 10  0.0625
00049F 01                          544          DC    X'01'          BASE 10  0.0625
0004A0 04                          545          DC    X'04'          BASE 10  0.2500
0004A1 04                          546          DC    X'04'          BASE 10  0.2500
0004A2 08                          547          DC    X'08'          BASE 10  0.5000
0004A3 08                          548          DC    X'08'          BASE 10  0.5000
0004A4 07                          549          DC    X'07'          BASE 10  0.4375
0004A5 0B                          550          DC    X'0B'          BASE 10  0.6875
```

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  LOC  OBJECT CODE    ADDR1 ADDR2  STMT   SOURCE STATEMENT

0004A6 09                           551           DC      X'09'           BASE 10  0.5625
0004A7 CC                           552           DC      X'0C'           BASE 10  0.7500
0004A8 0D                           553           DC      X'0D'           BASE 10  0.8125
0004A9 CE                           554           DC      X'0E'           BASE 10  0.8750
0004AA 10                           555           DC      X'10'           BASE 10  1.0000
0004AB 13                           556           DC      X'13'           BASE 10  1.1875
0004AC 02                           557           DC      X'02'           BASE 10  0.1250
0004AD 02                           558           DC      X'02'           BASE 10  0.1250
0004AE 02                           559           DC      X'02'           BASE 10  0.1250
0004AF 03                           560           DC      X'03'           BASE 10  0.1875
0004B0 03                           561           DC      X'03'           BASE 10  0.1875
0004B1 03                           562           DC      X'03'           BASE 10  0.1975
0004B2 05                           563           DC      X'05'           BASE 10  0.3125
0004B3 05                           564           DC      X'05'           BASE 10  0.3125
0004B4 C6                           565           DC      X'06'           BASE 10  0.3750
0004B5 06                           566           DC      X'06'           BASE 10  0.3750
0004B6 C7                           567           DC      X'07'           BASE 10  0.4375
0004B7 0A                           568           DC      X'0A'           BASE 10  0.6250
0004B8 0A                           569           DC      X'0A'           BASE 10  0.6250
0004B9 09                           570           DC      X'09'           BASE 10  0.5625
0004BA CF                           571           DC      X'0F'           BASE 10  0.9375
0004BB 14                           572           DC      X'14'           BASE 10  1.2500
0004BC 15                           573           DC      X'15'           BASE 10  1.3125
0004BD 11                           574           DC      X'11'           BASE 10  1.0625
0004BE 12                           575           DC      X'12'           BASE 10  1.1250
                                    576           END
0004C0 00000025                     577                   =F'37'
0004C4 000001C8                     578                   =F'456'
0004C8 00000F17                     579                   =F'3863'
0004CC 00000E37                     580                   =F'3639'
```

APPENDIX C

Assembler Listing of GEN2

```
  LOC     OBJECT CODE    ADDR1 ADDR2  STMT   SOURCE STATEMENT
 00000                                   1 GEN2     START 0
                                         2 ***********************************************************************
                                         3 * SOURCE---L.F.CANNON, UGA, DEPT. OF STAT., APR 1970                 *
                                         4 * PURPOSE--TO SUPPLY THE USER WITH A FAST PROCEDURE FOR GENERATING EX-*
                                         5 *          PONETIAL AND GAMMA DISTRIBUTED RANDOM NUMBERS.           *
                                         6 * USAGE----X=RANGAM(I,N). 'I' MUST BE AN ODD INTEGER. IT PRIMES THE  *
                                         7 *          GENERATING SCHEME. 'N' IS THE NUMBER OF EXPONETIAL RANDOM *
                                         8 *          NUMBERS SUMMED IN GENERATING A RANDOM NUMBER THAT IS GAMMA *
                                         9 *          (A=N,B=1).                                                *
                                        10 * EXAMPLE:X=RANGAM(55595,50)  'X' WILL BE GAMMA(A=50,B=1).           *
                                        11 * NOTE-----IF N=1 AN EXPONETIALLY DISTRIBUTED RANDOM NUMBER WILL BE  *
                                        12 *          GENERATED.                                                *
                                        13 * METHOD---THE METHOD IS THAT OF MARSAGLIA AS DESCRIBED IN  'COMMUNI- *
                                        14 *          CATIONS OF ACM',VOL.7,NO.5,MAY 1964. THE PROGRAM DOCUMEN- *
                                        15 *          TATION WILL FOLLOW THE NOTATION USED BY MARSAGLIA.        *
                                        16 ***********************************************************************
000008                                  17          ENTRY RANGAM         PROLOGUE, STANDARD FOR 360/65.
000000 07                               18          USING RANGAM,15
000001 40D9C1D5C7C1D4                   19          DC    XL1'07'
000008 90EC D00C            0000C       20          DC    CL7'RANGAM'
00000C 58B0 1004            00004       21 RANGAM   STM   14,12,12(13)
000010 58B0 B000            00000       22          L     11,4(,1)          LOAD ADRESS OF N INTO R11.
000014 41C0 0001            00001       23          L     11,0(,11)         LOAD N INTO R11
000018 41A0 0001            00001       24          LA    12,1              R12=1, USED AS AN INDEX.
00001C 3R00                             25          LA    10,1              R10=1, USED AS AN INCREMENT.
00001E 47F0 F08A            00092       26          SER   0,0               RESULT SUMMED IN FPR0.
000022 5C40 F148            00150       27 BRANCH   B     TEMP              EXECUTED ONLY ON THE FIRST CALL.
000026 5050 F140            00148       28          M     4,BPAT            OBTAIN NEW U.
00002A 3R22                             29          ST    5,LA              STORE U FOR NEXT GENERATION.
00002C 1866                             30          SER   2,2               SET FPR2=0, 'W'=0.
00002E 1844                             31          SR    6,6
000030 8D40 0006            00006       32 N1       SR    4,4               CLEAR 34.
000034 5940 F400            00408       33          SLDL  4,6
000038 47B0 F040            00048       34          C     4,=F'37'          TWO HIGH ORDER OCTAL DIGITS OF U IN R4(D1D2)
00003C 4364 F44A            00482       35          BC    11,NA             D1D2 COMPARED WITH 37(BASE 10)=45(BASE 8).
000040 8950 0006            00006       36          IC    6,D+294(4)        D1D2>=37 BRANCH NA.
000044 47F0 F06C            00074       37          SLL   5,6               OBTAIN DISCRETE RANDOM VARIBLE FROM D.
000048 8D40 0003            00003       38          BC    15,EX             R5=D5D6D7...
00004C 5940 F404            0040C       39 NA       SLDL  4,3               BRANCH EX.
000050 47B0 F058            000E0       40          C     4,=F'456'         R4=D1D2D3.
000054 4364 F2F4            002FC       41          BC    11,NB             456(BASE 10)=710(BASE 8).
000058 8950 0003            00003       42          IC    6,D-144(4)        D1D2D3>=456 BRANCH NB.
00005C 47F0 F06C            00074       43          SLL   5,3               OBTAIN DISCRETE RANDOM VARIABLE FROM D.
000060 8D40 0003            00003       44          BC    15,EX             R5=D5D6D7...
000064 5940 F408            004E0       45 NB       SLDL  4,3               BRANCH EX.
000068 47B0 F0A4            000AC       46          C     4,=F'3863'        R4=D1D2D3D4..
00006C 5B40 F40C            004E4       47          BC    11,NC             3863(BASE 10)=7427(BASE 8).
000070 4364 F384            0038C       48          S     4,=F'3639'        D1D2D3D4>=3863 BRANCH NC.
000074 1846                             49          IC    6,D(4)            OBTAIN PROPER ADDRESS FOR D TABLE.
000076 8C40 0010            00010       50 EX       LR    4,6               OBTAIN DISCRETE VARIABLE FROM D.
00007A 5050 F13C            00144       51 FL       SRDL  4,16              R4= DISCRETE VARIABLE.
00007E 9241 F13C      00144             52          ST    5,RES             R5= DISCRETE VARIABLE + HI ORDER 16 BITS OF
000082 7A20 F13C            00144       53          MVI   RES,X'41'         R5(NOW LO ORDER 16 BITS) WITH HI ORDER 8
000086 3A02                             54          AE    2,RES             BITS ZEROED OUT FOR CONVERSION TO FLOAT-
                                        55          AER   0,2               ING POINT. FPR2 CONTAINS RESULT + 'W'.
                                                                            FINAL RESULT SUMMED IN FPR0.
```

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  LOC  OBJECT CODE    ADDR1 ADDR2  STMT   SOURCE STATEMENT
000088 87CA F016            0001E   56             BXLE  12,10,BRANCH   PERFORM BRANCH LOOP N TIMES.
00008C 98EC D00C            0000C   57             LM    14,12,12(13)  RESTORE REGISTERS.
000090 07FE                         58             BCR   15,14       BRANCH TO MAIN LINE.
000092 5859 F140            00149   59 TEMP        L     5,LA         REPLACES INSTR. AT BRANCH.
000094 D203 F016 F08A 0001E 00092   60             MVC   BRANCH,TEMP  REPLACE BRANCH BY TEMP.
00009C 5851 1000            00000   61             L     5,0(,1)      ADDRESS OF PRIMER IN R5.
0000A0 5853 5000            00000   62             L     5,0(,5)      PRIMER IN R5.
0000A4 5051 F140            00148   63             ST    5,LA         STORE PRIMER FOR USE IN URAN.
0000A8 47F0 F01A            00022   64             B     BRANCH+4     BRANCH TO INSTRUCTION FOLLOWING 'BRANCH'.
0000AC 5851 F140            00148   65 NC          L     5,LA         RESTORE U TO R5.
0000B0 4120 0004            00004   66             LA    2,4
0000B4 1833                         67             SR    3,3
0000B6 5553 F36C            00374   68             CL    5,P(3)       U>=P(0) BRANCH ND.
0000BA 4780 F116            0011E   69             BC    11,ND
0000BE 1832                         70             SR    3,2
0000C0 1A32                         71 NE          AR    3,2          R3=IR(IN WORDS, 4 BYTES=1 WORD).
0000C2 5553 F16C            00174   72             CL    5,R(3)       U>=R(IR) BRANCH NF(CONTINUE LOOP).
0000C6 4730 F0B8            000C0   73             BC    11,NE
0000CA 5553 F26C            00274   74             CL    5,C(3)       C(IR)=<U<R(IR) BRANCH NF.
0000CE 4780 F0F0            000E8   75             BC    11,NF
0000D2 5C41 F148            00150   76             M     4,HPAT       OBTAIN NEW U.
0000D6 5C51 F140            00148   77             ST    5,LA         STORE FOR NEXT GENERATION.
0000DA 1B44                         78 GO          SR    4,4          CLEAR R4.
0000DC 8B30 0002            00002   79             SRL   3,2
0000E0 4343 F384            0038C   80             IC    4,D(3)       OBTAIN DISCRETE VARIABLE.
0000E4 47F0 F06E            00076   81             BC    15,FL
0000E8 5C41 F148            00150   82 NF          M     4,RPAT       OBTAIN NEW U.
0000EC 1982                         83             LR    8,2          USED FOR INDEXING.
0000EE 1A92                         84             AR    9,2          R8=IO.
0000F0 5558 F14C            00154   85             CL    5,O(8)       U>=O(IO) CONTINUE LOOP.
0000F4 47B0 F0F6            000FE   86             BC    11,*-4
0000F8 5C41 F148            00150   87             M     4,HPAT       OBTAIN NEW U.
0000FC 1B82                         88             SR    8,2
0000FE 1865                         89             LR    6,5
000100 1192                         90             LNR   9,2          R9=-4.
000102 1B82                         91             SR    8,2
000104 5C41 F148            00150   92 NH          M     4,HPAT       OBTAIN NEW U.
000108 1556                         93             CLR   5,6          R5>=R6 BRANCH TO CONTINUE.
00010A 4780 F108            00110   94             BC    11,*+6
00010E 1865                         95             LR    6,5          LOAD SMALLEST U IN R6.
000110 8689 F0FC            00104   96             BXH   8,9,NH       CONTINUE(R9 HAS BOTH INCREMENT & DELIMITER).
000114 5050 F140            00148   97             ST    5,LA         STORE FOR NEXT GENERATION.
000118 1856                         98             LR    5,6          R5=MIN(U1,U2,...,U(IO)).
00011A 47F0 F0D2            000DA   99 *           BC    15,GO
00011E 1A32                        100 ND          AR    3,2          R3=IP.
000120 5553 F36C            00374  101             CL    5,P(3)       U>=P(IP) CONTINUE LOOP.
000124 47B0 F116            0011E  102             BC    11,ND
000128 5030 F144            0014C  103             ST    3,J          STORE IP INTO J.
00012C 9245 F144      0014C        104             MVI   J,X'46'      CONVERT IP TO FLOATING POINT.
000130 7A20 F144            0014C  105             AE    2,J          NORMALIZED AND LOADED INTO FPR2 'M'.
000134 5C40 F148            00150  106             M     4,RPAT       OBTAIN NEW U.
000138 5050 F140            00148  107             ST    5,LA
00013C 5C46 F36C            00374  108             M     4,P(5)       U=U*P(0).
000140 47F0 F026            0002E  109             BC    15,N1        BRANCH N1.
000144                             110 RES         DS    1F
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  LOC  OBJECT CODE    ADDR1 ADDR2  STMT   SOURCE STATEMENT

000148                              111 LA      DS    1F
00014C                              112 J       DS    1F
000150 4BC27395                     113 BPAT    DC    XL4'4BC27395'
000154 00000000                     114 Q       DC    X'00000000'    BASE 10 0.0
000158 00000000                     115         DC    X'00000000'    BASE 10 0.0
00015C FA81CFA7                     116         DC    X'FA81CFA7'    BASE 10= 0.97927562333
000160 FFFAD848                     117         DC    X'FFFAD848'    BASE 10= 0.99967719382
000164 FFFFBC72                     118         DC    X'FFFFBC72'    BASE 10= 0.99999597364
000168 FFFFFF4C                     119         DC    X'FFFFFF4C'    BASE 10= 0.99999995912
00016C FFFFFFFF                     120         DC    X'FFFFFFFF'    BASE 10= 0.99999999963
000170 FFFFFFFF                     121         DC    X'FFFFFFFF'    BASE 10= 1.00000000000
000174 F1F24021                     122 A       DC    X'F1F24021'    BASE 10 0.94519942156
000178 F23EF28D                     123         DC    X'F23EF28D'    BASE 10 0.94628827211
00017C F28C810C                     124         DC    X'F28C810C'    BASE 10 0.94745647236
000180 F2C13730                     125         DC    X'F2C13730'    BASE 10 0.94826073593
000184 F2F0E518                     126         DC    X'F2F0E518'    BASE 10 0.94874248886
000188 F30742B9                     127         DC    X'F30742B9'    BASE 10 0.94932953916
00018C F317E405                     128         DC    X'F317E405'    BASE 10 0.94958329319
000190 F3261A5C                     129         DC    X'F3261A5C'    BASE 10 0.94983063691
000194 F32DF665                     130         DC    X'F32DF665'    BASE 10 0.94992008179
000198 F3B3051C                     131         DC    X'F3B3051C'    BASE 10 0.95199460160
00019C F42115AE                     132         DC    X'F42115AE'    BASE 10 0.95361457350
0001A0 F4B8D94F                     133         DC    X'F4B8D94F'    BASE 10 0.95525892423
0001A4 F4F4D2CF                     134         DC    X'F4F4D2CF'    BASE 10 0.95606066501
0001A8 F5502594                     135         DC    X'F5502594'    BASE 10 0.95839152226
0001AC F5521D7A                     136         DC    X'F5521D7A'    BASE 10 0.95974907159
0001B0 F6DA6B07                     137         DC    X'F6DA6B07'    BASE 10 0.96109651586
0001B4 F65F0CBE                     138         DC    X'F65F0CBE'    BASE 10 0.96237256579
0001B8 F6A471A9                     139         DC    X'F6A471A9'    BASE 10 0.96344671629
0001BC F6F9C8E1                     140         DC    X'F6F9C8E1'    BASE 10 0.96447806932
0001C0 F7290034                     141         DC    X'F7290034'    BASE 10 0.96548445247
0001C4 F761312A                     142         DC    X'F761312A'    BASE 10 0.96637678299
0001C8 F7941C9C                     143         DC    X'F7941C9C'    BASE 10 0.96710374973
0001CC F7C5B6EF                     144         DC    X'F7C5B6EF'    BASE 10 0.96796063535
0001D0 F7F1E012                     145         DC    X'F7F1E012'    BASE 10 0.96853524311
0001D4 F81CD87F                     146         DC    X'F81CD87F'    BASE 10 0.96920558791
0001D8 F843046B                     147         DC    X'F843046B'    BASE 10 0.96986129414
0001DC F871F734                     148         DC    X'F871F734'    BASE 10 0.97048062494
0001E0 F8970F94                     149         DC    X'F8970F94'    BASE 10 0.97116734391
0001E4 F8B90C57                     150         DC    X'F8B90C57'    BASE 10 0.97158600928
0001E8 F8DAC69F                     151         DC    X'F8DAC69F'    BASE 10 0.97209925453
0001EC F8F81060                     152         DC    X'F8F81060'    BASE 10 0.97258093522
0001F0 F91A2B9F                     153         DC    X'F91A2B9F'    BASE 10 0.97305557854
0001F4 F93A9989                     154         DC    X'F93A9989'    BASE 10 0.97352084759
0001F8 F9586658                     155         DC    X'F9586658'    BASE 10 0.97397651342
0001FC F9740441                     156         DC    X'F9740441'    BASE 10 0.97442652313
000200 F990DA1A                     157         DC    X'F990DA1A'    BASE 10 0.97485411775
000204 F9A34357                     158         DC    X'F9A34357'    BASE 10 0.97525425799
000208 F9C36880                     159         DC    X'F9C36880'    BASE 10 0.97563794277
00020C F9DC3773                     160         DC    X'F9DC3773'    BASE 10 0.97601648369
000210 F9F46745                     161         DC    X'F9F46745'    BASE 10 0.97638554092
000214 FACA550A                     162         DC    X'FACA550A'    BASE 10 0.9767201[illegible]
000218 FA20C498                     163         DC    X'FA20C498'    BASE 10 0.97705[illegible]
00021C FA33E7C0                     164         DC    X'FA33E7C0'    BASE 10 0.97735[illegible]
000220 FA4567F8                     165         DC    X'FA4567F8'    BASE 10 0.97763681231
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  LOC  OBJECT CODE    ADDR1 ADDR2  STMT   SOURCE STATEMENT

000224 FA590771                    166          DC     X'FA590771'    BASE 10 0.97791911485
000228 FA693F00                    167          DC     X'FA693F00'    BASE 10 0.97819899411
00022C FA706F58                    168          DC     X'FA706F58'    BASE 10 0.97847642599
000230 FA8FC18A                    169          DC     X'FA8FC18A'    BASE 10 0.97874079506
000234 FA9F650C                    170          DC     X'FA9F650C'    BASE 10 0.97899467042
000238 FAAF6AFF                    171          DC     X'FAAF6AFF'    BASE 10 0.97923992349
00023C FABFC9A2                    172          DC     X'FABFC9A2'    BASE 10 0.97947750293
000240 FACFA835                    173          DC     X'FACFA835'    BASE 10 0.97971601531
000244 FAD0203A                    174          DC     X'FAD0203A'    BASE 10 0.97993661339
000248 FAE0969A                    175          DC     X'FAE0969A'    BASE 10 0.98015717294
00024C FAF05A01                    176          DC     X'FAF05A01'    BASE 10 0.98036925689
000250 FB050436                    177          DC     X'FB050436'    BASE 10 0.98057295141
000254 FB143589                    178          DC     X'FB143589'    BASE 10 0.98077711647
000258 FB1F2994                    179          DC     X'FB1F2994'    BASE 10 0.98096419111
00025C FB2910CA                    180          DC     X'FB2910CA'    BASE 10 0.98109613594
000260 FB32FF11                    181          DC     X'FB32FF11'    BASE 10 0.98124587942
000264 FB3CA379                    182          DC     X'FB3CA379'    BASE 10 0.98139602139
000268 FB44A894                    183          DC     X'FB44A894'    BASE 10 0.98151466399
00026C FB4C437C                    184          DC     X'FB4C437C'    BASE 10 0.98163744057
000270 FB4FAA91                    185          DC     X'FB4FAA91'    BASE 10 0.98168461111
000274 F17705FD                    186 C        DC     X'F17705FD'    BASE 10 0.94327490089
000278 F1FF3941                    187          DC     X'F1FF3941'    BASE 10 0.94528534518
00027C F24FC2F8                    188          DC     X'F24FC2F8'    BASE 10 0.94651430666
000280 F294C937                    189          DC     X'F294C937'    BASE 10 0.94757244939
000284 F2CC0889                    190          DC     X'F2CC0889'    BASE 10 0.94863921067
000288 F2F44595                    191          DC     X'F2F44595'    BASE 10 0.94907597217
00028C F3136BCA                    192          DC     X'F3136BCA'    BASE 10 0.94951534456
000290 F31C973D                    193          DC     X'F31C973D'    BASE 10 0.94967253451
000294 F329C2C7                    194          DC     X'F329C2C7'    BASE 10 0.94985597071
000298 F33CA878                    195          DC     X'F33CA878'    BASE 10 0.95114449779
00029C F3839726                    196          DC     X'F3839726'    BASE 10 0.95195110838
0002A0 F426207D                    197          DC     X'F426207D'    BASE 10 0.95370563497
0002A4 F495FC59                    198          DC     X'F495FC59'    BASE 10 0.95549929478
0002A8 F4F09646                    199          DC     X'F4F09646'    BASE 10 0.95693234495
0002AC F5504921                    200          DC     X'F5504921'    BASE 10 0.95845135294
0002B0 F58FF212                    201          DC     X'F58FF212'    BASE 10 0.95996010742
0002B4 F60FC556                    202          DC     X'F60FC556'    BASE 10 0.96115288522
0002B8 F65F7807                    203          DC     X'F65F7807'    BASE 10 0.96237915935
0002BC F6AF0971                    204          DC     X'F6AF0971'    BASE 10 0.96359307254
0002C0 F6F39363                    205          DC     X'F6F39363'    BASE 10 0.96455302830
0002C4 F72F0177                    206          DC     X'F72F0177'    BASE 10 0.96554574146
0002C8 F7643697                    207          DC     X'F7643697'    BASE 10 0.96637007930
0002CC F7A299F7                    208          DC     X'F7A299F7'    BASE 10 0.96732353403
0002D0 F7C77089                    209          DC     X'F7C77089'    BASE 10 0.96788774270
0002D4 F7FFCF83                    210          DC     X'F7FFCF83'    BASE 10 0.96873186740
0002D8 F823618C                    211          DC     X'F823618C'    BASE 10 0.96923987709
0002DC F84A0A08                    212          DC     X'F84A0A08'    BASE 10 0.96987974944
0002E0 F87C7A4C                    213          DC     X'F87C7A4C'    BASE 10 0.97064928261
0002E4 F8A485E7                    214          DC     X'F8A485E7'    BASE 10 0.97125042281
0002E8 F8C7F209                    215          DC     X'F8C7F209'    BASE 10 0.97180092549
0002EC F8F15379                    216          DC     X'F8F15379'    BASE 10 0.97218820298
0002F0 F8FD0158                    217          DC     X'F8FD0158'    BASE 10 0.97261055441
0002F4 F921F2CE                    218          DC     X'F921F2CE'    BASE 10 0.97317426251
0002F8 F93F5998                    219          DC     X'F93F5998'    BASE 10 0.97360757562
0002FC F9663071                    220          DC     X'F9663071'    BASE 10 0.97421630871
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  LOC  OBJECT CODE   ADDR1 ADDR2  STMT   SOURCE STATEMENT

000300 F9755995                    221        DC    X'F9755995'    BASE 10 0.97458619708
000304 F9989911                    222        DC    X'F9989911'    BASE 10 0.97503048590
000308 F9B7401A                    223        DC    X'F9B7401A'    BASE 10 0.97545242919
00030C F9CC980C                    224        DC    X'F9CC980C'    BASE 10 0.97577828454
000310 F9E059C9                    225        DC    X'F9E059C9'    BASE 10 0.97607968974
000314 FA0040C4                    226        DC    X'FA0040C4'    BASE 10 0.97656636056
000318 FA12E797                    227        DC    X'FA12E797'    BASE 10 0.97695777372
00031C FA2E054F                    228        DC    X'FA2E054F'    BASE 10 0.97727717817
000320 FA3976D5                    229        DC    X'FA3976D5'    BASE 10 0.97743933414
000324 FA52557F                    230        DC    X'FA52557F'    BASE 10 0.97761881658
000328 FA6393A8                    231        DC    X'FA6393A8'    BASE 10 0.97816016935
00032C FA73F667                    232        DC    X'FA73F667'    BASE 10 0.97833194768
000330 FA8355F5                    233        DC    X'FA8355F5'    BASE 10 0.97854652118
000334 FA99E954                    234        DC    X'FA99E954'    BASE 10 0.97884151993
000338 FAA9798A                    235        DC    X'FAA9798A'    BASE 10 0.97913322195
00033C FAB9172F                    236        DC    X'FAB9172F'    BASE 10 0.97941727549
000340 FAC5C620                    237        DC    X'FAC5C620'    BASE 10 0.97958029049
000344 FAD74018                    238        DC    X'FAD74018'    BASE 10 0.97985429215
000348 FAE43406                    239        DC    X'FAE43406'    BASE 10 0.98004665327
00034C FAF1937C                    240        DC    X'FAF1937C'    BASE 10 0.98024848031
000350 FB146F59                    241        DC    X'FB146F59'    BASE 10 0.98053642181
000354 FB115246                    242        DC    X'FB115246'    BASE 10 0.98073305333
000358 FB144DE4                    243        DC    X'FB144DE4'    BASE 10 0.98079947595
00035C FB2456F4                    244        DC    X'FB2456F4'    BASE 10 0.98102349782
000360 FB2FA960                    245        DC    X'FB2FA960'    BASE 10 0.98119594912
000364 FB36E548                    246        DC    X'FB36E548'    BASE 10 0.98139639999
000368 FB40D549                    247        DC    X'FB40D549'    BASE 10 0.98145902546
00036C FB498D04                    248        DC    X'FB498D04'    BASE 10 0.98159104699
000370 FB4C07A1                    249        DC    X'FB4C07A1'    BASE 10 0.98163745593
000374 FB4FAA91                    250 P      DC    X'FB4FAA91'    BASE 10 0.98168436111
000378 FFEA03DE                    251        DC    X'FFEA03DE'    BASE 10 0.99964453737
00037C FFFF08EA                    252        DC    X'FFFF08EA'    BASE 10 0.99998385579
000380 FFFFFE1C                    253        DC    X'FFFFFE1C'    BASE 10 0.99999988746
000384 FFFFFFF7                    254        DC    X'FFFFFFF7'    BASE 10 0.99999999794
000388 FFFFFFFF                    255        DC    X'FFFFFFFF'    BASE 10 0.99999999996
00038C 00                          256 D      DC    X'00'          BASE 10  0.0
00038D 0A                          257        DC    X'0A'          BASE 10  0.6250
00038E 0B                          258        DC    X'0B'          BASE 10  0.6875
00038F 10                          259        DC    X'10'          BASE 10  1.0000
000390 15                          260        DC    X'15'          BASE 10  1.3125
000391 20                          261        DC    X'20'          BASE 10  2.0000
000392 35                          262        DC    X'35'          BASE 10  3.3125
000393 2B                          263        DC    X'2B'          BASE 10  2.6875
000394 36                          264        DC    X'36'          BASE 10  3.3750
000395 01                          265        DC    X'01'          BASE 10  0.0625
000396 02                          266        DC    X'02'          BASE 10  0.1250
000397 03                          267        DC    X'03'          BASE 10  0.1875
000398 05                          268        DC    X'05'          BASE 10  0.3125
000399 04                          269        DC    X'04'          BASE 10  0.2500
00039A 06                          270        DC    X'06'          BASE 10  0.3750
00039B 08                          271        DC    X'08'          BASE 10  0.5000
00039C 07                          272        DC    X'07'          BASE 10  0.4375
00039D 09                          273        DC    X'09'          BASE 10  0.5625
00039E 0C                          274        DC    X'0C'          BASE 10  0.7500
00039F 0D                          275        DC    X'0D'          BASE 10  0.8125
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LOC  OBJECT CODE    ADDR1 ADDR2  STMT   SOURCE STATEMENT

0003A0 0E                          276          DC    X'0E'           BASE 10  0.8750
0003A1 0F                          277          DC    X'0F'           BASE 10  0.9375
0003A2 14                          278          DC    X'14'           BASE 10  1.2500
0003A3 11                          279          DC    X'11'           BASE 10  1.0625
0003A4 16                          280          DC    X'16'           BASE 10  1.3750
0003A5 13                          281          DC    X'13'           BASE 10  1.1875
0003A6 12                          282          DC    X'12'           BASE 10  1.1250
0003A7 18                          283          DC    X'18'           BASE 10  1.5000
0003A8 1C                          284          DC    X'1C'           BASE 10  1.7500
0003A9 1E                          285          DC    X'1E'           BASE 10  1.8750
0003AA 19                          286          DC    X'19'           BASE 10  1.5625
0003AB 17                          287          DC    X'17'           BASE 10  1.4375
0003AC 1B                          288          DC    X'1B'           BASE 10  1.6875
0003AD 1A                          289          DC    X'1A'           BASE 10  1.6250
0003AE 23                          290          DC    X'23'           BASE 10  2.1875
0003AF 1F                          291          DC    X'1F'           BASE 10  1.9375
0003B0 22                          292          DC    X'22'           BASE 10  2.1250
0003B1 25                          293          DC    X'25'           BASE 10  2.3125
0003B2 21                          294          DC    X'21'           BASE 10  2.0625
0003B3 1D                          295          DC    X'1D'           BASE 10  1.8125
0003B4 28                          296          DC    X'28'           BASE 10  2.5000
0003B5 30                          297          DC    X'30'           BASE 10  3.0000
0003B6 33                          298          DC    X'33'           BASE 10  3.1875
0003B7 24                          299          DC    X'24'           BASE 10  2.2500
0003B8 2F                          300          DC    X'2F'           BASE 10  2.9375
0003B9 3E                          301          DC    X'3E'           BASE 10  3.8750
0003BA 29                          302          DC    X'29'           BASE 10  2.5625
0003BB 26                          303          DC    X'26'           BASE 10  2.3750
0003BC 39                          304          DC    X'39'           BASE 10  3.5625
0003BD 2E                          305          DC    X'2E'           BASE 10  2.8750
0003BE 37                          306          DC    X'37'           BASE 10  3.4375
0003BF 2A                          307          DC    X'2A'           BASE 10  2.6250
0003C0 32                          308          DC    X'32'           BASE 10  3.1250
0003C1 2D                          309          DC    X'2D'           BASE 10  2.8125
0003C2 2C                          310          DC    X'2C'           BASE 10  2.7500
0003C3 3F                          311          DC    X'3F'           BASE 10  3.9375
0003C4 3C                          312          DC    X'3C'           BASE 10  3.7500
0003C5 27                          313          DC    X'27'           BASE 10  2.4375
0003C6 34                          314          DC    X'34'           BASE 10  3.2500
0003C7 3A                          315          DC    X'3A'           BASE 10  3.6250
0003C8 31                          316          DC    X'31'           BASE 10  3.0625
0003C9 38                          317          DC    X'38'           BASE 10  3.5000
0003CA 3D                          318          DC    X'3D'           BASE 10  3.8125
0003CB 3B                          319          DC    X'3B'           BASE 10  3.6875
0003CC 02                          320          DC    X'02'           BASE 10  0.1250
0003CD 03                          321          DC    X'03'           BASE 10  0.1875
0003CE 03                          322          DC    X'03'           BASE 10  0.1875
0003CF 03                          323          DC    X'03'           BASE 10  0.1875
0003D0 03                          324          DC    X'03'           BASE 10  0.1875
0003D1 03                          325          DC    X'03'           BASE 10  0.1875
0003D2 03                          326          DC    X'03'           BASE 10  0.1875
0003D3 05                          327          DC    X'05'           BASE 10  0.3125
0003D4 0F                          328          DC    X'0F'           BASE 10  0.9375
0003D5 0F                          329          DC    X'0F'           BASE 10  0.9375
0003D6 14                          330          DC    X'14'           BASE 10  1.2500
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  LOC  OBJECT CODE   ADDR1 ADDR2  STMT   SOURCE STATEMENT

0003D7 14                         331          DC      X'14'        BASE 10  1.2500
0003D8 14                         332          DC      X'14'        BASE 10  1.2500
0003D9 12                         333          DC      X'12'        BASE 10  1.1250
0003DA 12                         334          DC      X'12'        BASE 10  1.1250
0003DB 12                         335          DC      X'12'        BASE 10  1.1250
0003DC 12                         336          DC      X'12'        BASE 10  1.1250
0003DD 1A                         337          DC      X'1A'        BASE 10  1.6250
0003DE 25                         338          DC      X'25'        BASE 10  2.3125
0003DF 25                         339          DC      X'25'        BASE 10  2.3125
0003E0 25                         340          DC      X'25'        BASE 10  2.3125
0003E1 25                         341          DC      X'25'        BASE 10  2.3125
0003E2 21                         342          DC      X'21'        BASE 10  2.0625
0003E3 21                         343          DC      X'21'        BASE 10  2.0625
0003E4 1D                         344          DC      X'1D'        BASE 10  1.8125
0003E5 1D                         345          DC      X'1D'        BASE 10  1.8125
0003E6 30                         346          DC      X'30'        BASE 10  3.0000
0003E7 2F                         347          DC      X'2F'        BASE 10  2.9375
0003E8 2F                         348          DC      X'2F'        BASE 10  2.9375
0003E9 3E                         349          DC      X'3E'        BASE 10  3.8750
0003EA 3E                         350          DC      X'3E'        BASE 10  3.8750
0003EB 3E                         351          DC      X'3E'        BASE 10  3.8750
0003EC 26                         352          DC      X'26'        BASE 10  2.3750
0003ED 26                         353          DC      X'26'        BASE 10  2.3750
0003EE 26                         354          DC      X'26'        BASE 10  2.3750
0003EF 39                         355          DC      X'39'        BASE 10  3.5625
0003F0 39                         356          DC      X'39'        BASE 10  3.5625
0003F1 39                         357          DC      X'39'        BASE 10  3.5625
0003F2 39                         358          DC      X'39'        BASE 10  3.5625
0003F3 39                         359          DC      X'39'        BASE 10  3.5625
0003F4 2E                         360          DC      X'2E'        BASE 10  2.8750
0003F5 2E                         361          DC      X'2E'        BASE 10  2.8750
0003F6 2E                         362          DC      X'2E'        BASE 10  2.8750
0003F7 37                         363          DC      X'37'        BASE 10  3.4375
0003F8 37                         364          DC      X'37'        BASE 10  3.4375
0003F9 37                         365          DC      X'37'        BASE 10  3.4375
0003FA 37                         366          DC      X'37'        BASE 10  3.4375
0003FB 37                         367          DC      X'37'        BASE 10  3.4375
0003FC 37                         368          DC      X'37'        BASE 10  3.4375
0003FD 2D                         369          DC      X'2D'        BASE 10  2.8125
0003FE 2D                         370          DC      X'2D'        BASE 10  2.8125
0003FF 2D                         371          DC      X'2D'        BASE 10  2.8125
000400 2D                         372          DC      X'2D'        BASE 10  2.8125
000401 2C                         373          DC      X'2C'        BASE 10  2.7500
000402 2C                         374          DC      X'2C'        BASE 10  2.7500
000403 2C                         375          DC      X'2C'        BASE 10  2.7500
000404 2C                         376          DC      X'2C'        BASE 10  2.7500
000405 2C                         377          DC      X'2C'        BASE 10  2.7500
000406 3F                         378          DC      X'3F'        BASE 10  3.9375
000407 3F                         379          DC      X'3F'        BASE 10  3.9375
000408 3F                         380          DC      X'3F'        BASE 10  3.9375
000409 3C                         381          DC      X'3C'        BASE 10  3.7500
00040A 3C                         382          DC      X'3C'        BASE 10  3.7500
00040B 3C                         383          DC      X'3C'        BASE 10  3.7500
00040C 3C                         384          DC      X'3C'        BASE 10  3.7500
00040D 27                         385          DC      X'27'        BASE 10  2.4375
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  LOC  OBJECT CODE    ADDR1 ADDR2  STMT   SOURCE STATEMENT

00040E 27                           386          DC     X'27'          BASE 10  2.4375
00040F 3A                           387          DC     X'3A'          BASE 10  3.6250
000410 3A                           388          DC     X'3A'          BASE 10  3.6250
000411 3A                           389          DC     X'3A'          BASE 10  3.6250
000412 3A                           390          DC     X'3A'          BASE 10  3.6250
000413 3A                           391          DC     X'3A'          BASE 10  3.6250
000414 31                           392          DC     X'31'          BASE 10  3.0625
000415 38                           393          DC     X'38'          BASE 10  3.5000
000416 38                           394          DC     X'38'          BASE 10  3.5000
000417 38                           395          DC     X'38'          BASE 10  3.5000
000418 38                           396          DC     X'38'          BASE 10  3.5000
000419 38                           397          DC     X'38'          BASE 10  3.5000
00041A 38                           398          DC     X'38'          BASE 10  3.5000
00041B 3D                           399          DC     X'3D'          BASE 10  3.8125
00041C 3D                           400          DC     X'3D'          BASE 10  3.8125
00041D 3D                           401          DC     X'3D'          BASE 10  3.8125
00041E 3D                           402          DC     X'3D'          BASE 10  3.8125
00041F 3B                           403          DC     X'3B'          BASE 10  3.6875
000420 3B                           404          DC     X'3B'          BASE 10  3.6875
000421 3B                           405          DC     X'3B'          BASE 10  3.6875
000422 3B                           406          DC     X'3B'          BASE 10  3.6875
000423 3B                           407          DC     X'3B'          BASE 10  3.6875
000424 02                           408          DC     X'02'          BASE 10  0.1250
000425 02                           409          DC     X'02'          BASE 10  0.1250
000426 05                           410          DC     X'05'          BASE 10  0.3125
000427 05                           411          DC     X'05'          BASE 10  0.3125
000428 05                           412          DC     X'05'          BASE 10  0.3125
000429 05                           413          DC     X'05'          BASE 10  0.3125
00042A 05                           414          DC     X'05'          BASE 10  0.3125
00042B 04                           415          DC     X'04'          BASE 10  0.2500
00042C 04                           416          DC     X'04'          BASE 10  0.2500
00042D 06                           417          DC     X'06'          BASE 10  0.3750
00042E 06                           418          DC     X'06'          BASE 10  0.3750
00042F 06                           419          DC     X'06'          BASE 10  0.3750
000430 06                           420          DC     X'06'          BASE 10  0.3750
000431 07                           421          DC     X'07'          BASE 10  0.4375
000432 07                           422          DC     X'07'          BASE 10  0.4375
000433 0D                           423          DC     X'0D'          BASE 10  0.8125
000434 0E                           424          DC     X'0E'          BASE 10  0.8750
000435 0E                           425          DC     X'0E'          BASE 10  0.8750
000436 0E                           426          DC     X'0E'          BASE 10  0.8750
000437 0F                           427          DC     X'0F'          BASE 10  0.9375
000438 0F                           428          DC     X'0F'          BASE 10  0.9375
000439 0F                           429          DC     X'0F'          BASE 10  0.9375
00043A 11                           430          DC     X'11'          BASE 10  1.0625
00043B 11                           431          DC     X'11'          BASE 10  1.0625
00043C 16                           432          DC     X'16'          BASE 10  1.3750
00043D 16                           433          DC     X'16'          BASE 10  1.3750
00043E 16                           434          DC     X'16'          BASE 10  1.3750
00043F 12                           435          DC     X'12'          BASE 10  1.1250
000440 18                           436          DC     X'18'          BASE 10  1.5000
000441 18                           437          DC     X'18'          BASE 10  1.5000
000442 18                           438          DC     X'18'          BASE 10  1.5000
000443 18                           439          DC     X'18'          BASE 10  1.5000
000444 1E                           440          DC     X'1E'          BASE 10  1.8750
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  LOC  OBJECT CODE    ADDR1 ADDR2  STMT   SOURCE STATEMENT

000445 1E                           441          DC    X'1E'        BASE 10  1.8750
000446 1F                           442          DC    X'1F'        BASE 10  1.8750
000447 19                           443          DC    X'19'        BASE 10  1.5625
000448 1B                           444          DC    X'1B'        BASE 10  1.6875
000449 1B                           445          DC    X'1B'        BASE 10  1.6875
00044A 1B                           446          DC    X'1B'        BASE 10  1.6875
00044B 1A                           447          DC    X'1A'        BASE 10  1.6250
00044C 1A                           448          DC    X'1A'        BASE 10  1.6250
00044D 1A                           449          DC    X'1A'        BASE 10  1.6250
00044E 1A                           450          DC    X'1A'        BASE 10  1.6250
00044F 1A                           451          DC    X'1A'        BASE 10  1.6250
000450 23                           452          DC    X'23'        BASE 10  2.1875
000451 1F                           453          DC    X'1F'        BASE 10  1.9375
000452 22                           454          DC    X'22'        BASE 10  2.1250
000453 22                           455          DC    X'22'        BASE 10  2.1250
000454 22                           456          DC    X'22'        BASE 10  2.1250
000455 25                           457          DC    X'25'        BASE 10  2.3125
000456 25                           458          DC    X'25'        BASE 10  2.3125
000457 21                           459          DC    X'21'        BASE 10  2.0625
000458 21                           460          DC    X'21'        BASE 10  2.0625
000459 21                           461          DC    X'21'        BASE 10  2.0625
00045A 1D                           462          DC    X'1D'        BASE 10  1.8125
00045B 1D                           463          DC    X'1D'        BASE 10  1.8125
00045C 1D                           464          DC    X'1D'        BASE 10  1.8125
00045D 1D                           465          DC    X'1D'        BASE 10  1.8125
00045E 28                           466          DC    X'28'        BASE 10  2.5000
00045F 28                           467          DC    X'28'        BASE 10  2.5000
000460 30                           468          DC    X'30'        BASE 10  3.0000
000461 2F                           469          DC    X'2F'        BASE 10  2.9375
000462 29                           470          DC    X'29'        BASE 10  2.5625
000463 26                           471          DC    X'26'        BASE 10  2.3750
000464 26                           472          DC    X'26'        BASE 10  2.3750
000465 2E                           473          DC    X'2E'        BASE 10  2.8750
000466 32                           474          DC    X'32'        BASE 10  3.1250
000467 2D                           475          DC    X'2D'        BASE 10  2.8125
000468 2C                           476          DC    X'2C'        BASE 10  2.7500
000469 27                           477          DC    X'27'        BASE 10  2.4375
00046A 27                           478          DC    X'27'        BASE 10  2.4375
00046B 31                           479          DC    X'31'        BASE 10  3.0625
00046C 00                           480          DC    X'00'        BASE 10  0.0000
00046D 00                           481          DC    X'00'        BASE 10  0.0000
00046E 00                           482          DC    X'00'        BASE 10  0.0000
00046F 01                           483          DC    X'01'        BASE 10  0.0625
000470 04                           484          DC    X'04'        BASE 10  0.2500
000471 04                           485          DC    X'04'        BASE 10  0.2500
000472 04                           486          DC    X'04'        BASE 10  0.2500
000473 0B                           487          DC    X'0B'        BASE 10  0.6875
000474 0B                           488          DC    X'0B'        BASE 10  0.6875
000475 0B                           489          DC    X'0B'        BASE 10  0.6875
000476 0B                           490          DC    X'0B'        BASE 10  0.6875
000477 0B                           491          DC    X'0B'        BASE 10  0.6875
000478 0B                           492          DC    X'0B'        BASE 10  0.6875
000479 0C                           493          DC    X'0C'        BASE 10  0.7500
00047A 0C                           494          DC    X'0C'        BASE 10  0.7500
00047B 0C                           495          DC    X'0C'        BASE 10  0.7500
```

LOC	OBJECT CODE	ADDR1	ADDR2	STMT	SOURCE STATEMENT		
00047C	0C			496	DC	X'0C'	BASE 10 0.7500
00047D	0C			497	DC	X'0C'	BASE 10 0.7500
00047E	0D			498	DC	X'0D'	BASE 10 0.8125
00047F	0D			499	DC	X'0D'	BASE 10 0.8125
000480	0D			500	DC	X'0D'	BASE 10 0.8125
000481	10			501	DC	X'10'	BASE 10 1.0000
000482	10			502	DC	X'10'	BASE 10 1.0000
000483	16			503	DC	X'16'	BASE 10 1.3750
000484	16			504	DC	X'16'	BASE 10 1.3750
000485	16			505	DC	X'16'	BASE 10 1.3750
000486	16			506	DC	X'16'	BASE 10 1.3750
000487	18			507	DC	X'18'	BASE 10 1.5000
000488	18			508	DC	X'18'	BASE 10 1.5000
000489	1C			509	DC	X'1C'	BASE 10 1.7500
00048A	1C			510	DC	X'1C'	BASE 10 1.7500
00048B	1C			511	DC	X'1C'	BASE 10 1.7500
00048C	1C			512	DC	X'1C'	BASE 10 1.7500
00048D	1C			513	DC	X'1C'	BASE 10 1.7500
00048E	1E			514	DC	X'1E'	BASE 10 1.8750
00048F	19			515	DC	X'19'	BASE 10 1.5625
000490	19			516	DC	X'19'	BASE 10 1.5625
000491	19			517	DC	X'19'	BASE 10 1.5625
000492	19			518	DC	X'19'	BASE 10 1.5625
000493	19			519	DC	X'19'	BASE 10 1.5625
000494	17			520	DC	X'17'	BASE 10 1.4375
000495	17			521	DC	X'17'	BASE 10 1.4375
000496	17			522	DC	X'17'	BASE 10 1.4375
000497	17			523	DC	X'17'	BASE 10 1.4375
000498	17			524	DC	X'17'	BASE 10 1.4375
000499	17			525	DC	X'17'	BASE 10 1.4375
00049A	17			526	DC	X'17'	BASE 10 1.4375
00049B	1B			527	DC	X'1B'	BASE 10 1.6875
00049C	1B			528	DC	X'1B'	BASE 10 1.6875
00049D	23			529	DC	X'23'	BASE 10 2.1875
00049E	23			530	DC	X'23'	BASE 10 2.1875
00049F	1F			531	DC	X'1F'	BASE 10 1.9375
0004A0	1F			532	DC	X'1F'	BASE 10 1.9375
0004A1	1F			533	DC	X'1F'	BASE 10 1.9375
0004A2	20			534	DC	X'20'	BASE 10 2.0000
0004A3	20			535	DC	X'20'	BASE 10 2.0000
0004A4	20			536	DC	X'20'	BASE 10 2.0000
0004A5	20			537	DC	X'20'	BASE 10 2.0000
0004A6	33			538	DC	X'33'	BASE 10 3.1875
0004A7	24			539	DC	X'24'	BASE 10 2.2500
0004A8	24			540	DC	X'24'	BASE 10 2.2500
0004A9	24			541	DC	X'24'	BASE 10 2.2500
0004AA	29			542	DC	X'29'	BASE 10 2.5625
0004AB	35			543	DC	X'35'	BASE 10 3.3125
0004AC	2A			544	DC	X'2A'	BASE 10 2.6250
0004AD	2A			545	DC	X'2A'	BASE 10 2.6250
0004AE	2B			546	DC	X'2B'	BASE 10 2.6875
0004AF	2B			547	DC	X'2B'	BASE 10 2.6875
0004B0	34			548	DC	X'34'	BASE 10 3.2500
0004B1	36			549	DC	X'36'	BASE 10 3.3750
0004B2	00			550	DC	X'00'	BASE 10 0.0000

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  LOC  OBJECT CODE    ADDR1 ADDR2  STMT   SOURCE STATEMENT

0004B3 00                           551          DC     X'00'          BASE 10  0.0000
0004B4 00                           552          DC     X'00'          BASE 10  0.0000
0004B5 01                           553          DC     X'01'          BASE 10  0.0625
0004B6 01                           554          DC     X'01'          BASE 10  0.0625
0004B7 01                           555          DC     X'01'          BASE 10  0.0625
0004B8 04                           556          DC     X'04'          BASE 10  0.2500
0004B9 04                           557          DC     X'04'          BASE 10  0.2500
0004BA 08                           558          DC     X'08'          BASE 10  0.5000
0004BB 08                           559          DC     X'08'          BASE 10  0.5000
0004BC 07                           560          DC     X'07'          BASE 10  0.4375
0004BD 0B                           561          DC     X'0B'          BASE 10  0.6875
0004BE 09                           562          DC     X'09'          BASE 10  0.5625
0004BF 0C                           563          DC     X'0C'          BASE 10  0.7500
0004C0 0D                           564          DC     X'0D'          BASE 10  0.8125
0004C1 0E                           565          DC     X'0E'          BASE 10  0.8750
0004C2 10                           566          DC     X'10'          BASE 10  1.0000
0004C3 13                           567          DC     X'13'          BASE 10  1.1875
0004C4 02                           568          DC     X'02'          BASE 10  0.1250
0004C5 02                           569          DC     X'02'          BASE 10  0.1250
0004C6 02                           570          DC     X'02'          BASE 10  0.1250
0004C7 03                           571          DC     X'03'          BASE 10  0.1875
0004C8 03                           572          DC     X'03'          BASE 10  0.1875
0004C9 03                           573          DC     X'03'          BASE 10  0.1875
0004CA 05                           574          DC     X'05'          BASE 10  0.3125
0004CB 05                           575          DC     X'05'          BASE 10  0.3125
0004CC 06                           576          DC     X'06'          BASE 10  0.3750
0004CD 06                           577          DC     X'06'          BASE 10  0.3750
0004CE 07                           578          DC     X'07'          BASE 10  0.4375
0004CF 0A                           579          DC     X'0A'          BASE 10  0.6250
0004D0 0A                           580          DC     X'0A'          BASE 10  0.6250
0004D1 09                           581          DC     X'09'          BASE 10  0.5625
0004D2 0F                           582          DC     X'0F'          BASE 10  0.9375
0004D3 14                           583          DC     X'14'          BASE 10  1.2500
0004D4 15                           584          DC     X'15'          BASE 10  1.3125
0004D5 11                           585          DC     X'11'          BASE 10  1.0625
0004D6 12                           586          DC     X'12'          BASE 10  1.1250
                                    587          END
0004D8 00000025                     588                 =F'37'
0004DC 000001C8                     589                 =F'456'
0004E0 00000F17                     590                 =F'3863'
0004E4 00000E37                     591                 =F'3639'
```

APPENDIX D

Assembler Listing of GEN3

NOT REPRODUCIBLE

```
                                                                                        PAGE    1
  LOC  OBJECT CODE    ADDR1 ADDR2  STMT   SOURCE STATEMENT                          F28APR70   6/30/71
000000                               1 GEN3     START 0
                                     2          PRINT ON,NOGEN,NODATA
                                     3 ***************************************************************
                                     4 * SOURCE---L.E.CANNON, UGA, DEPT. OF STAT., AUG 1970           *
                                     5 * PURPOSE--TO SUPPLY THE USER WITH A FAST PROCEDURE FOR GENERATING NOR*
                                     6 *          MALLY DISTRIBUTED RANDOM NUMBERS.                    *
                                     7 * USAGE----X=RANORM(IOD). 'IOD' MUST BE AN ODD POSITIVE INTEGER, IT *
                                     8 *          PRIMES THE GENERATING SCHEME.                        *
                                     9 * EXAMPLE--X=RANORM(56865).                                     *
                                    10 * METHOD---THE METHOD IS THAT OF MARSAGLIA AS DESCRIBED IN 'COMMUNI- *
                                    11 *          CATIONS OF ACM',VOL.7,NO.1,JAN 1964. THE PROGRAM DOCUMEN- *
                                    12 *          TATION WILL FOLLOW THE NOTATION USED BY MARSAGLIA.   *
                                    13 ***************************************************************
                                    14          ENTRY RANORM
000000 47F0 F00C           0000C    15 RANORM   B     12(0,15)
000004 C7                           16          DC    XL1'07'
000005 40D9C1D5D6D9D4               17          DC    CL7' RANORM'
00000C 90E8 D00C           0000C    18          STM   14,8,12(13)
000010 0580                         19          BALR  8,0
000012                              20          USING *,8
000012 185D                         21          LR    5,13
000014 41D0 82B2           007C4    22          LA    13,AREA
000018 50D0 5008           00008    23          ST    13,8(0,5)
00001C 505D 0004           00004    24          ST    5,4(0,13)
000020 47F0 806E           000B0    25 BRANCH   B     TEMP
000024 1B66                         26          SR    6,6
000026 5C40 82FA           0030C    27          M     4,BPAT         GENERATE U(UNIFORM ON (0,1)).
00002A 5050 82FE           00310    28          ST    5,LA
00002E 3B00                         29          SER   0,0            RESULT TO BE PLACED IN FPR0.
000030 1B44                         30          SR    4,4
000032 8D40 0006           00006    31          SLDL  4,6            D1D2 IN R4.
000036 5540 866E           006C0    32          CL    4,=F'44'       44(BASE 10)=54(BASE 8).
00003A 47B0 8038           0004A    33          BC    11,L1          IF D1D2>=44 BRANCH TO L1.
00003E 8950 0006           00006    34          SLL   5,6            R5=D3D4D5D6D7...
000042 4364 85A6           00598    35          IC    6,A(4)         OBTAIN TABLE VALUE.
000046 47F0 8064           00076    36          B     EX
00004A 8D40 0003           00003    37 L1       SLDL  4,3            R4=D1D2D3.
00004E 5540 86B2           006C4    38          CL    4,=F'475'      475(BASE 10)=733(BASE 8).
000052 47B0 8050           00062    39          BC    11,L2          IF D1D2D3>=475 BRANCH TO L2.
000056 8950 0003           00003    40          SLL   5,3            R5=D5D6D7...
00005A 4364 8460           00472    41          IC    6,A-326(4)     OBTAIN TABLE VALUE.
00005E 47F0 8064           00076    42          B     EX
000062 8D40 0003           00003    43 L2       SLDL  4,3            R4=D1D2D3D4.
000066 5540 86B6           006C8    44          CL    4,=F'3561'     3961(BASE 10)=7571(BASE 8).
00006A 47B0 8098           000CA    45          BC    11,L3          IF D1D2D3D4>=3961 BRANCH TO L3.
00006E 5B40 86BA           006CC    46          S     4,=F'3703'     OBTAIN PROPER TABLE ADDRESS.
000072 4364 85A6           00588    47          IC    6,A(4)         OBTAIN TABLE VALUE.
000076 1846                         48 EX       LR    4,6            R4=DISCRETE TABLE VALUE.
000078 8C40 0010           00010    49 FL       SRDL  4,16           R5=TABLE R.V. WITH D5D6.. TACKED ON.
00007C 5050 8302           00314    50          ST    5,RES          CONVERT TO FLOATING POINT.
000080 5850 82FE           00310    51          L     5,LA           ATTACH + OR -.
000084 5C40 82FA           0030C    52          M     4,HPAT
000088 5050 82FE           00310    53          ST    5,LA
00008C 1255                         54          LTR   5,5
00008E 4720 8088           0009A    55          BP    *+12
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  LOC  OBJECT CODE    ADDR1 ADDR2  STMT   SOURCE STATEMENT

000092 92C1 8302      C0314          56          MVI   RES,X'C1'
000096 47F0 909C            0009F    57          BC    15,*+8
00009A 9241 8302      00314          58          MVI   RES,X'41'
00009E 7A00 8302            00314    59          AE    0,RES        NORMALIZED RESULT IN FPRO.
0000A2 58D0 92B6            002C8    60 OUT      L     13,ARFA+4
0000A6 98E5 D00C            0000C    61          LM    14,5,12(13)
0000AA 92FF D00C      0000C          62          MVI   12(13),X'FF'
0000AE 07FE                          63          BCR   15,14
0000B0 5850 92FE            00310    64 TEMP     L     5,LA         REPLACES INSTR. AT BRANCH.
0000B4 D203 909E 809E 00020 000B0    65          MVC   BRANCH,TEMP
0000BA 5850 1000            00000    66          L     5,0(,1)
0000BE 5850 5000            00000    67          L     5,0(,5)
0000C2 5050 92FE            00310    68          ST    5,LA
0000C6 47F0 9012            00024    69          B     BRANCH+4
0000CA 5850 92FE            00310    70 L3       L     5,LA         R5=U.
0000CE 4120 0004            00004    71          LA    2,4
0000D2 5550 969E            00600    72          CL    5,=X'FF4F10F0'
0000D6 47B0 91BE            00100    73          BC    11,L4        IF U>=FF4F10F0(BASE 16) GO TO L4.
0000DA 1162                          74          LNR   6,2
0000DC 1A62                          75          AR    6,2          INCREMENT I(R6).
0000DE 5556 83C6            00308    76          CL    5,B(6)       COMPARE U TO B(I).
0000E2 47B0 90CA            000DC    77          BC    11,*-6       IF U>=B(I) CONTINUE LOOP.
0000E6 5556 8486            00498    78          CL    5,C(6)       COMPARE U TO C(I).
0000EA 47B0 90FC            000FE    79          BC    11,L8        IF C(I)=<U<B(I) GO TO L8.
0000EE 5C40 82FA            003CC    80          M     4,BPAT       GENERATE A NEW U.
0000F2 5050 92FE            00310    81          ST    5,LA
0000F6 8860 0002            00002    82          SRL   6,2
0000FA 47F0 91B4            001C6    83          B     L5
0000FE 5C40 92FA            003CC    84 L8       M     4,BPAT       GENERATE U1.
000102 1F75                          85          LR    7,5          SAVE U1.
000104 5C40 92FA            003CC    86          M     4,BPAT       GENERATE U2.
000108 5050 92FE            00310    87          ST    5,LA
00010C 1575                          88          CLR   7,5
00010E 47B0 9106            00118    89          BC    11,L6        R5=MIN(U1,U2).
000112 1E57                          90          LR    5,7
000114 5870 92FE            00310    91          L     7,LA
000118 8860 0001            00001    92 L6       SRL   6,1
00011C 1B44                          93          SR    4,4
00011E 4846 8546            00558    94          LH    4,D(6)
000122 1024                          95          LPR   2,4
000124 1837                          96          LR    3,7
000126 8830 0010            00010    97          SRL   3,16
00012A 1532                          98          CLR   3,2
00012C 47B0 9126            00138    99          BC    11,L9        IF MAX(U1,U2)<D(I) GO TO L5.
000130 8860 0001            00001   100          SRL   6,1
000134 47F0 91B4            001C6   101          B     L5
000138 8850 0001            00001   102 L9       SRL   6,1
00013C 5050 929A            002AC   103          ST    5,MIN
000140 1F75                         104          SLR   7,5          R7=MAX-MIN.
000142 8870 0008            00008   105          SRL   7,8
000146 5070 829E            00280   106          ST    7,DIF        CONVERT TO FLOATING POINT.
00014A 9240 829E      00280         107          MVI   DIF,X'40'
00014E 3822                         108          SER   2,2
000150 7A20 829E            00280   109          AE    2,DIF
000154 8850 000C            0000C   110          SRL   5,12
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 LOC   OBJECT CODE   ADDR1 ADDR2  STMT   SOURCE STATEMENT                                        FORAPR70   6/30/71

000158 5050 82A2           00284   111          ST    5,W            CONVERT TO FLOATING POINT.
00015C 9240 82A2     00284         112          MVI   W,X'40'
000160 7840 86C2           006D4   113          LE    4,=E'0.0625'
000164 3864                        114          LER   6,4
000166 7B40 82A2           00284   115          SE    4,W
00016A 7A60 82A2           00284   116          AE    6,W
00016E 1B33                        117          SR    3,3
000170 4336 867E           00690   118          IC    3,A+216(6)     R3=TABLE RANDOM VARIABLE.
000174 8930 0001           00001   119          SLL   3,1            R3=R3*2.
000178 5030 82A2           00284   120          ST    3,W
00017C 9245 82A2     00284         121          MVI   W,X'45'
000180 7A60 82A2           00284   122          AE    6,W
000184 3C46                        123          MER   4,6
000186 7C40 86C6           006D8   124          ME    4,=E'0.5'      FPR4=-(MIN/16-1/16)*(2*A(I)+MIN/16+1/16)*.5
00018A 7040 82A2           00284   125          STE   4,W
                                   126          CALL  EXP,(W)        CALCULATE EXP(W).
0001A6 7B00 86CA           006DC   138          SE    0,=E'1.0'      FPR0=EXP(W)-1.
0001AA 8960 0002           00002   139          SLL   6,2
0001AE 7C06 8306           00318   140          ME    0,E(6)         FPR0=(EXP(W)-1)*E(I).
0001B2 5850 82FE           00310   141          L     5,LA
0001B6 3902                        142          CER   0,2            COMPERE FPR0 TO DIF.
0001B8 47D0 80EC           001FE   143          BC    13,L8          IF FPR0<=DIF GO TO L9.
0001BC 5850 829A           002AC   144          L     5,MIN
0001C0 3B02                        145          SER   0,2
0001C2 8860 0002           00002   146          SRL   6,2
0001C6 1B44                        147 L5       SR    4,4
0001C8 4346 867E           00690   148          IC    4,A+216(6)     OBTAIN RANDOM VARIABLE.
0001CC 47F0 8064           00078   149          B     FL
0001D0 5850 82FE           00310   150 L4       L     5,LA
0001D4 1B77                        151          SR    7,7
0001D6 5C40 82FA           0030C   152          M     4,RPAT         OBTAIN U1.
0001DA 1B44                        153          SR    4,4
0001DC 5050 82FE           00310   154          ST    5,LA
0001E0 8850 0008           00008   155          SRL   5,8            PREPARE FOR FLT. PT. CONVERSION.
0001E4 5050 8246           00288   156          ST    5,U1
0001E8 5850 82FE           00310   157          L     5,LA
0001EC 1C45                        158          MR    4,5            R4=U1*U1.
0001EE 1E74                        159          ALR   7,4
0001F0 5850 82FE           00310   160          L     5,LA
0001F4 5C40 82FA           0030C   161          M     4,RPAT         OBTAIN U2.
0001F8 5050 82FE           00310   162          ST    5,LA
0001FC 1B44                        163          SR    4,4
0001FE 8850 0008           00008   164          SRL   5,8
000202 5050 82AA           0028C   165          ST    5,U2
000206 5850 82FE           00310   166          L     5,LA
00020A 1C45                        167          MR    4,5            R4=U2*U2.
00020C 1E74                        168          ALR   7,4            R7=U1**2+U2**2.
00020E 4710 81BE           001D0   169          BO    L4             IF U1**2+U2**2>=1 BRANCH TO L4.
000212 8870 0008           00008   170          SRL   7,8
000216 5070 82AE           002C0   171          ST    7,S            CONVERT TO FLT. PT.
00021A 9240 82AE     002C0         172          MVI   S,X'40'
00021E 7A00 82AE           002C0   173          AE    0,S
000222 7000 82AE           002C0   174          STE   0,S
                                   175          CALL  ALOG,(S)       CALCULATE LN(U1**2+U2**2).
00023E 7C00 86CE           006E0   187          ME    0,=E'-2.0'
```

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  LOC  OBJECT CODE    ADDR1 ADDR2  STMT   SOURCE STATEMENT

000242 7A00 8A02            006F4   188            AE    0,=E'9.0'
000246 7000 92AE            002C0   189            DE    0,S
00024A 7000 82AE            002C0   190            STE   0,S
                                    191            CALL  SQ- -,(S)
000266 9240 92A6      002B8         203            MVI   U1,X'40'       CONVERT TO FLT. PT.
00026A 3922                         204            SER   2,2
00026C 7A20 92A6            002B8   205            AE    2,U1
000270 3C20                         206            MER   2,0
000272 7920 8606            006F8   207            CE    2,=E'3.0'
000276 4720 827E            00290   208            BC    2,L10          IF FPR2>3.0 BRANCH TO L10.
00027A 9240 92AA      002BC         209            MVI   U2,X'40'       CONVERT TO FLT. PT.
00027E 3922                         210            SER   2,2
000280 7A20 92AA            002BC   211            AE    2,U2
000284 3C20                         212            MER   2,0
000286 3900                         213            SER   0,0
000288 7920 8606            006F8   214            CE    2,=E'3.0'
00028C 47D0 818E            001D0   215            BC    13,L4          IF FPR2=<3.0 BRANCH TO L4.
000290 58F0 82FE            00310   216 L10        L     5,LA
000294 5C40 82FA            0030C   217            M     4,BPAT
000298 5050 82FE            00310   218            ST    5,LA
00029C 1255                         219            LTR   5,5
00029E 4720 9292            002A4   220            BP    *+6
0002A2 3122                         221            LNER  2,2
0002A4 3802                         222            LER   0,2
0002A6 47F0 8090            000A2   223            B     OUT
0002AC                              224 MIN        DS    1F
0002B0                              225 DIF        DS    1F
0002B4                              226 W          DS    1F
0002B8                              227 U1         DS    1F
0002BC                              228 U2         DS    1F
0002C0                              229 S          DS    1F
0002C4                              230 AREA       DS    18F
00030C 48C27395                     231 BPAT       DC    XL4'48C27395'
000310                              232 LA         DS    1F
000314                              233 RES        DS    1F
000318 42100000                     234 E          DC    E'16.0'
00031C 41D59373                     235            DC    E'13.34385943'
000320 42138138                     236            DC    E'19.69230768'
000324 42111111                     237            DC    E'17.06666666'
000328 41F225F6                     238            DC    E'14.13426759'
00032C 41C42720                     239            DC    E'12.63455216'
000330 42124925                     240            DC    E'18.28571428'
000334 41F0540F                     241            DC    E'15.02052273'
000338 41A66996                     242            DC    E'10.40126568'
00033C 41B69FA4                     243            DC    E'11.41397461'
000340 42155555                     244            DC    E'21.33333333'
000344 4198C29F                     245            DC    E'9.54715094'
000348 421C71C7                     246            DC    E'28.44444444'
00034C 419F4FAF                     247            DC    E'9.95653629'
000350 42174501                     248            DC    E'23.27272727'
000354 42249249                     249            DC    E'36.57142856'
000358 42200000                     250            DC    E'32.0'
00035C 4192B43C                     251            DC    E'9.16902116'
000360 41870E22                     252            DC    E'8.49131562'
000364 41801682                     253            DC    E'8.81641004'
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  LOC  OBJECT CODE    ADDR1 ADDR2  STMT   SOURCE STATEMENT

000368 422AAAAB                     254          DC     E'42.66666666'
00036C 417A321F                     255          DC     E'7.637235440'
000370 41830046                     256          DC     E'8.187568839'
000374 42333333                     257          DC     E'51.19999999'
000378 417E7[illegible]             258          DC     E'7.903368655'
00037C 4172716E                     259          DC     E'7.152692877'
000380 42400000                     260          DC     E'64.0'
000384 415D59BE                     261          DC     E'5.834165752'
000388 416869C3                     262          DC     E'6.525576115'
00038C 416EF77F                     263          DC     E'6.931516923'
000390 42800000                     264          DC     E'128.0'
000394 4162970F                     265          DC     E'6.161878474'
000398 415444CF                     266          DC     E'5.267287374'
00039C 43100000                     267          DC     E'256.0'
0003A0 41656C02                     268          DC     E'6.331869363'
0003A4 4155612B                     269          DC     E'5.352210855'
0003A8 415A2A33                     270          DC     E'5.635302894'
0003AC 41599D04                     271          DC     E'5.537357695'
0003B0 4150C3FD                     272          DC     E'5.020504743'
0003B4 41524243                     273          DC     E'5.141177028'
0003B8 414F7A55                     274          DC     E'4.954927053'
0003BC 41B5F9A1                     275          DC     E'11.59447734'
0003C0 42555555                     276          DC     E'85.33333333'
0003C4 4219999A                     277          DC     E'25.6'
0003C8 41AE22A0                     278          DC     E'10.65510165'
0003CC 41763333                     279          DC     E'7.397499660'
0003D0 416B94AB                     280          DC     E'6.722816549'
0003D4 415FE6F0                     281          DC     E'5.993865632'
0003D8 F7DC23B8                     282 8        DC     X'F7DC23B8'
0003DC F8272283                     283          DC     X'F8272283'
0003E0 F86F9303                     284          DC     X'F86F9303'
0003E4 F8B7C400                     285          DC     X'F8B7C400'
0003E8 F8FBFEA0                     286          DC     X'F8FBFEA0'
0003EC F93F3937                     287          DC     X'F93F3937'
0003F0 F98D7426                     288          DC     X'F98D7426'
0003F4 F9C0C5F0                     289          DC     X'F9C0C5F0'
0003F8 F9FA50DE                     290          DC     X'F9FA50DE'
0003FC FA3541DE                     291          DC     X'FA3541DE'
000400 FA717F89                     292          DC     X'FA717F89'
000404 FAA74087                     293          DC     X'FAA74087'
000408 FAD03982                     294          DC     X'FAD03982'
00040C FB115AA5                     295          DC     X'FB115AA5'
000410 FB4C190E                     296          DC     X'FB4C190E'
000414 FB7CF968                     297          DC     X'FB7CF968'
000418 FBAC6582                     298          DC     X'FBAC6582'
00041C FBDA168C                     299          DC     X'FBDA168C'
000420 FC072AD8                     300          DC     X'FC072AD8'
000424 FC31807D                     301          DC     X'FC31807D'
000428 FC59C042                     302          DC     X'FC59C042'
00042C FC83BF9A                     303          DC     X'FC83BF9A'
000430 FCA60FE7                     304          DC     X'FCA60FE7'
000434 FCCAEEF0                     305          DC     X'FCCAEEF0'
000438 FCEE547E                     306          DC     X'FCEE547E'
00043C FD0B990F                     307          DC     X'FD0B990F'
000440 FD31F952                     308          DC     X'FD31F952'
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LCC	OBJECT CODE	ADDR1	ADDR2	STMT		SOURCE	STATEMENT
000444	FD46D39F			309		DC	X'FD46D39F'
000448	FD593D30			310		DC	X'FD593D30'
00044C	FD79158F			311		DC	X'FD79158F'
000450	FD8E5D36			312		DC	X'FD8E5D36'
000454	FDA339E6			313		DC	X'FDA339E6'
000458	FDB7A0AF			314		DC	X'FDB7A0AF'
00045C	FDC9A2EA			315		DC	X'FDC9A2EA'
000460	FDD96E0E			316		DC	X'FDD96E0E'
000464	FDECA777			317		DC	X'FDECA777'
000468	FDFD0A1D			318		DC	X'FDFD0A1D'
00046C	FE0DE049			319		DC	X'FE0DE049'
000470	FF1F0F0C			320		DC	X'FF1F0F0C'
000474	FF273FF6			321		DC	X'FF273FF6'
000478	FF2F27B3			322		DC	X'FF2F27B3'
00047C	FF60B811			323		DC	X'FF60B811'
000480	FE89939C			324		DC	X'FE89939C'
000484	FFB03425			325		DC	X'FFB03425'
000488	FFF0414D			326		DC	X'FFF0414D'
00048C	FF1CC2A3			327		DC	X'FF1CC2A3'
000490	FF33C0E3			328		DC	X'FF33C0E3'
000494	FF4F10F0			329		DC	X'FF4F10F0'
000498	F7C1E758			330	C	DC	X'F7C1E758'
00049C	F8197296			331		DC	X'F8197296'
0004A0	F8623897			332		DC	X'F8623897'
0004A4	F8AD10F0			333		DC	X'F8AD10F0'
0004A8	F8F4401B			334		DC	X'F8F4401B'
0004AC	F936F652			335		DC	X'F936F652'
0004B0	F974C777			336		DC	X'F974C777'
0004B4	F99E47A2			337		DC	X'F99E47A2'
0004B8	F9F359C8			338		DC	X'F9F359C8'
0004BC	FA2FA596			339		DC	X'FA2FA596'
0004C0	FA6F93C1			340		DC	X'FA6F93C1'
0004C4	FA9FA0C6			341		DC	X'FA9FA0C6'
0004C8	FAD762AF			342		DC	X'FAD762AF'
0004CC	FB0D2A1F			343		DC	X'FB0D2A1F'
0004D0	FB4707AD			344		DC	X'FB4707AD'
0004D4	FB731C08			345		DC	X'FB731C08'
0004D8	FBA89731			346		DC	X'FBA89731'
0004DC	FBD726FA			347		DC	X'FBD726FA'
0004E0	FBFE098F			348		DC	X'FBFE098F'
0004E4	FC2FC406			349		DC	X'FC2FC406'
0004E8	FC53839A			350		DC	X'FC53839A'
0004EC	FC764456			351		DC	X'FC764456'
0004F0	FCA2B2A2			352		DC	X'FCA2B2A2'
0004F4	FCC29AB5			353		DC	X'FCC29AB5'
0004F8	FCDA1FBA			354		DC	X'FCDA1FBA'
0004FC	FD0531CD			355		DC	X'FD0531CD'
000500	FD2263B2			356		DC	X'FD2263B2'
000504	FD3C3FE6			357		DC	X'FD3C3FE6'
000508	FD57C534			358		DC	X'FD57C534'
00050C	FD6F65A6			359		DC	X'FD6F65A6'
000510	FD83A96A			360		DC	X'FD83A96A'
000514	FD99AD8B			361		DC	X'FD99AD8B'
000518	FDA934FA			362		DC	X'FDA934FA'
00051C	FDB8E0C9			363		DC	X'FDB8E0C9'

LOC	OBJECT CODE	ADDR1	ADDR2	STMT	SOURCE STATEMENT	
000520	FD0B8064			364	DC	X'FD0B8064'
000524	FDF22BFA			365	DC	X'FDF22BFA'
000528	FDF59092			366	DC	X'FDF59092'
00052C	FF05A299			367	DC	X'FF05A299'
000530	FE1229E8			368	DC	X'FE1229E8'
000534	FE220621			369	DC	X'FE220621'
000538	FE2C0971			370	DC	X'FE2C0971'
00053C	FE6B50B8			371	DC	X'FE6B50B8'
000540	FE7F231D			372	DC	X'FE7F231D'
000544	FEBCE8B6			373	DC	X'FEBCE8B6'
000548	FEF23046			374	DC	X'FEF23046'
00054C	FF1732E6			375	DC	X'FF1732E6'
000550	FF2F9EC3			376	DC	X'FF2F9EC3'
000554	FF3F796D			377	DC	X'FF3F796D'
000558	FF7D			378	DC	X'FF7D'
00055A	FF3B			379	DC	X'FF3B'
00055C	FC29			380	DC	X'FC29'
00055E	FF75			381	DC	X'FF75'
000560	FF7D			382	DC	X'FF7D'
000562	FEF9			383	DC	X'FEF9'
000564	FD70			384	DC	X'FD70'
000566	FFBE			385	DC	X'FFBE'
000568	FE35			386	DC	X'FE35'
00056A	FEB8			387	DC	X'FEB8'
00056C	FAF1			388	DC	X'FAF1'
00056E	FDF3			389	DC	X'FDF3'
000570	F5C2			390	DC	X'F5C2'
000572	FE35			391	DC	X'FE35'
000574	F953			392	DC	X'F953'
000576	F0A3			393	DC	X'F0A3'
000578	F374			394	DC	X'F374'
00057A	F0B2			395	DC	X'F0B2'
00057C	FD70			396	DC	X'FD70'
00057E	FDB2			397	DC	X'FDB2'
000580	FD07			398	DC	X'FD07'
000582	FCED			399	DC	X'FCED'
000584	FD2F			400	DC	X'FD2F'
000586	F831			401	DC	X'F831'
000588	FD2F			402	DC	X'FD2F'
00058A	FCAC			403	DC	X'FCAC'
00058C	F189			404	DC	X'F189'
00058E	FBA5			405	DC	X'FBA5'
000590	FC29			406	DC	X'FC29'
000592	FC6A			407	DC	X'FC6A'
000594	FCC3			408	DC	X'C0C3'
000596	FBE7			409	DC	X'FBE7'
000598	FB22			410	DC	X'FB22'
00059A	B00C			411	DC	X'B00C'
00059C	FC29			412	DC	X'FC29'
00059E	FB22			413	DC	X'FB22'
0005A0	FB64			414	DC	X'FB64'
0005A2	FB64			415	DC	X'FB64'
0005A4	FA9F			416	DC	X'FA9F'
0005A6	FAE1			417	DC	X'FAE1'
0005A8	FA9F			418	DC	X'FA9F'

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  LOC  OBJECT CODE    ADDR1 ADDR2  STMT   SOURCE STATEMENT

0005AA FEF9                         419          DC     X'FEF9'
0005AC D645                         420          DC     X'D645'
0005AE F68D                         421          DC     X'F68D'
0005B0 FE76                         422          DC     X'FE76'
0005B2 FCAC                         423          DC     X'FCAC'
0005B4 FCAA                         424          DC     X'FCAA'
0005B6 FB45                         425          DC     X'FB45'
0005B8 00                           426 A        DC     X'00'
0005B9 00                           427          DC     X'00'
0005BA 00                           428          DC     X'00'
0005BB 01                           429          DC     X'01'
0005BC 01                           430          DC     X'01'
0005BD 01                           431          DC     X'01'
0005BE 02                           432          DC     X'02'
0005BF 02                           433          DC     X'02'
0005C0 03                           434          DC     X'03'
0005C1 03                           435          DC     X'03'
0005C2 03                           436          DC     X'03'
0005C3 04                           437          DC     X'04'
0005C4 04                           438          DC     X'04'
0005C5 04                           439          DC     X'04'
0005C6 08                           440          DC     X'08'
0005C7 08                           441          DC     X'08'
0005C8 0C                           442          DC     X'0C'
0005C9 0D                           443          DC     X'0D'
0005CA 0D                           444          DC     X'0D'
0005CB 0E                           445          DC     X'0E'
0005CC 0F                           446          DC     X'0F'
0005CD 13                           447          DC     X'13'
0005CE 14                           448          DC     X'14'
0005CF 15                           449          DC     X'15'
0005D0 16                           450          DC     X'16'
0005D1 17                           451          DC     X'17'
0005D2 02                           452          DC     X'02'
0005D3 05                           453          DC     X'05'
0005D4 05                           454          DC     X'05'
0005D5 06                           455          DC     X'06'
0005D6 05                           456          DC     X'05'
0005D7 07                           457          DC     X'07'
0005D8 07                           458          DC     X'07'
0005D9 09                           459          DC     X'09'
0005DA 09                           460          DC     X'09'
0005DB 0A                           461          DC     X'0A'
0005DC 0A                           462          DC     X'0A'
0005DD 0B                           463          DC     X'0B'
0005DE 0B                           464          DC     X'0B'
0005DF 0C                           465          DC     X'0C'
0005E0 0F                           466          DC     X'0F'
0005E1 10                           467          DC     X'10'
0005E2 11                           468          DC     X'11'
0005E3 12                           469          DC     X'12'
0005E4 06                           470          DC     X'06'
0005E5 06                           471          DC     X'06'
0005E6 06                           472          DC     X'06'
0005E7 06                           473          DC     X'06'
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LOC	OBJECT CODE	ADDR1	ADDR2	STMT	SOURCE STATEMENT	
0005E8	06			474	DC	X'06'
0005E9	07			475	DC	X'07'
0005EA	07			476	DC	X'07'
0005EB	07			477	DC	X'07'
0005EC	09			478	DC	X'09'
0005ED	09			479	DC	X'09'
0005EE	09			480	DC	X'09'
0005EF	0A			481	DC	X'0A'
0005F0	0A			482	DC	X'0A'
0005F1	0F			483	DC	X'0F'
0005F2	0F			484	DC	X'0F'
0005F3	0F			485	DC	X'0F'
0005F4	0F			486	DC	X'0F'
0005F5	0F			487	DC	X'0F'
0005F6	10			488	DC	X'10'
0005F7	10			489	DC	X'10'
0005F8	10			490	DC	X'10'
0005F9	10			491	DC	X'10'
0005FA	11			492	DC	X'11'
0005FB	11			493	DC	X'11'
0005FC	11			494	DC	X'11'
0005FD	16			495	DC	X'16'
0005FE	18			496	DC	X'18'
0005FF	18			497	DC	X'18'
000600	18			498	DC	X'18'
000601	18			499	DC	X'18'
000602	18			500	DC	X'18'
000603	18			501	DC	X'18'
000604	19			502	DC	X'19'
000605	1A			503	DC	X'1A'
000606	1A			504	DC	X'1A'
000607	1A			505	DC	X'1A'
000608	1A			506	DC	X'1A'
000609	1A			507	DC	X'1A'
00060A	1A			508	DC	X'1A'
00060B	1B			509	DC	X'1B'
00060C	1B			510	DC	X'1B'
00060D	1B			511	DC	X'1B'
00060E	1B			512	DC	X'1B'
00060F	1D			513	DC	X'1D'
000610	1D			514	DC	X'1D'
000611	1F			515	DC	X'1F'
000612	20			516	DC	X'20'
000613	20			517	DC	X'20'
000614	20			518	DC	X'20'
000615	22			519	DC	X'22'
000616	23			520	DC	X'23'
000617	23			521	DC	X'23'
000618	27			522	DC	X'27'
000619	00			523	DC	X'00'
00061A	01			524	DC	X'01'
00061B	05			525	DC	X'05'
00061C	05			526	DC	X'05'
00061D	05			527	DC	X'05'
00061E	05			528	DC	X'05'

LOC	OBJECT CODE	ADDR1	ADDR2	STMT	SOURCE STATEMENT	
00061F	05			529	DC	X'05'
000620	07			530	DC	X'07'
000621	08			531	DC	X'08'
000622	08			532	DC	X'08'
000623	08			533	DC	X'08'
000624	08			534	DC	X'09'
000625	09			535	DC	X'09'
000626	0B			536	DC	X'0B'
000627	0C			537	DC	X'0C'
000628	0D			538	DC	X'0D'
000629	0F			539	DC	X'0F'
00062A	10			540	DC	X'10'
00062B	11			541	DC	X'11'
00062C	12			542	DC	X'12'
00062D	12			543	DC	X'12'
00062E	12			544	DC	X'12'
00062F	13			545	DC	X'13'
000630	13			546	DC	X'13'
000631	13			547	DC	X'13'
000632	14			548	DC	X'14'
000633	14			549	DC	X'14'
000634	15			550	DC	X'15'
000635	18			551	DC	X'18'
000636	19			552	DC	X'19'
000637	19			553	DC	X'19'
000638	19			554	DC	X'19'
000639	19			555	DC	X'19'
00063A	19			556	DC	X'19'
00063B	1B			557	DC	X'1B'
00063C	1C			558	DC	X'1C'
00063D	1C			559	DC	X'1C'
00063E	1C			560	DC	X'1C'
00063F	1C			561	DC	X'1C'
000640	1D			562	DC	X'1D'
000641	1D			563	DC	X'1D'
000642	1E			564	DC	X'1E'
000643	1F			565	DC	X'1F'
000644	1F			566	DC	X'1E'
000645	1F			567	DC	X'1F'
000646	1F			568	DC	X'1F'
000647	21			569	DC	X'21'
000648	21			570	DC	X'21'
000649	22			571	DC	X'22'
00064A	24			572	DC	X'24'
00064B	25			573	DC	X'25'
00064C	26			574	DC	X'26'
00064D	00			575	DC	X'00'
00064E	03			576	DC	X'03'
00064F	03			577	DC	X'03'
000650	03			578	DC	X'03'
000651	03			579	DC	X'03'
000652	04			580	DC	X'04'
000653	07			581	DC	X'07'
000654	07			582	DC	X'07'
000655	0D			583	DC	X'0D'

LOC	OBJECT CODE	ADDR1	ADDR2	STMT	SOURCE STATEMENT	
000656	0E			584	DC	X'0E'
000657	0E			585	DC	X'0E'
000658	0F			586	DC	X'0F'
000659	10			587	DC	X'10'
00065A	10			588	DC	X'10'
00065B	11			589	DC	X'11'
00065C	11			590	DC	X'11'
00065D	13			591	DC	X'13'
00065E	14			592	DC	X'14'
00065F	14			593	DC	X'14'
000660	14			594	DC	X'14'
000661	15			595	DC	X'15'
000662	15			596	DC	X'15'
000663	15			597	DC	X'15'
000664	15			598	DC	X'15'
000665	15			599	DC	X'15'
000666	17			600	DC	X'17'
000667	18			601	DC	X'18'
000668	18			602	DC	X'18'
000669	18			603	DC	X'18'
00066A	18			604	DC	X'18'
00066B	1C			605	DC	X'1C'
00066C	1C			606	DC	X'1C'
00066D	1E			607	DC	X'1E'
00066E	1E			608	DC	X'1E'
00066F	1E			609	DC	X'1E'
000670	21			610	DC	X'21'
000671	21			611	DC	X'21'
000672	24			612	DC	X'24'
000673	24			613	DC	X'24'
000674	24			614	DC	X'24'
000675	24			615	DC	X'24'
000676	25			616	DC	X'25'
000677	25			617	DC	X'25'
000678	28			618	DC	X'28'
000679	28			619	DC	X'28'
00067A	28			620	DC	X'28'
00067B	28			621	DC	X'28'
00067C	28			622	DC	X'28'
00067D	28			623	DC	X'28'
00067E	29			624	DC	X'29'
00067F	29			625	DC	X'29'
000680	29			626	DC	X'29'
000681	29			627	DC	X'29'
000682	29			628	DC	X'29'
000683	2A			629	DC	X'2A'
000684	2A			630	DC	X'2A'
000685	2A			631	DC	X'2A'
000686	2A			632	DC	X'2A'
000687	2B			633	DC	X'2B'
000688	2B			634	DC	X'2B'
000689	2B			635	DC	X'2B'
00068A	2C			636	DC	X'2C'
00068B	2C			637	DC	X'2C'
00068C	2D			638	DC	X'2D'

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  LOC  OBJECT CODE    ADDR1 ADDR2  STMT   SOURCE STATEMENT
00068D 20                           639           DC    X'20'
00068E 2E                           640           DC    X'2E'
00068F 2F                           641           DC    X'2F'
000690 0F                           642           DC    X'0F'
000691 12                           643           DC    X'12'
000692 0C                           644           DC    X'0C'
000693 0E                           645           DC    X'0E'
000694 11                           646           DC    X'11'
000695 13                           647           DC    X'13'
000696 0D                           648           DC    X'0D'
000697 10                           649           DC    X'10'
000698 17                           650           DC    X'17'
000699 15                           651           DC    X'15'
00069A 08                           652           DC    X'08'
00069B 19                           653           DC    X'19'
00069C 09                           654           DC    X'09'
00069D 1B                           655           DC    X'1B'
00069E 0A                           656           DC    X'0A'
00069F 06                           657           DC    X'06'
0006A0 07                           658           DC    X'07'
0006A1 14                           659           DC    X'14'
0006A2 1E                           660           DC    X'1E'
0006A3 18                           661           DC    X'18'
0006A4 05                           662           DC    X'05'
0006A5 1F                           663           DC    X'1F'
0006A6 1D                           664           DC    X'1D'
0006A7 04                           665           DC    X'04'
0006A8 1E                           666           DC    X'1E'
0006A9 21                           667           DC    X'21'
0006AA 03                           668           DC    X'03'
0006AB 28                           669           DC    X'28'
0006AC 24                           670           DC    X'24'
0006AD 22                           671           DC    X'22'
0006AE 01                           672           DC    X'01'
0006AF 26                           673           DC    X'26'
0006B0 2C                           674           DC    X'2C'
0006B1 00                           675           DC    X'00'
0006B2 25                           676           DC    X'25'
0006B3 2B                           677           DC    X'2B'
0006B4 29                           678           DC    X'29'
0006B5 2A                           679           DC    X'2A'
0006B6 2E                           680           DC    X'2E'
0006B7 2D                           681           DC    X'2D'
0006B8 2F                           682           DC    X'2F'
0006B9 14                           683           DC    X'14'
0006BA 02                           684           DC    X'02'
0006BB 09                           685           DC    X'09'
0006BC 16                           686           DC    X'16'
0006BD 20                           687           DC    X'20'
0006BE 23                           688           DC    X'23'
0006BF 27                           689           DC    X'27'
                                    690           END
0006C0 0000092C                     691                 =F'44'
0006C4 00000109                     692                 =F'475'
0006C8 00000F79                     693                 =F'3961'
```

LOC	OBJECT CODE	ADDR1	ADDR2	STMT	SOURCE STATEMENT
0006CC	00000E77			694	=F'3703'
0006D0	FF4F10F0			695	=X'FF4F10F0'
0006D4	40100000			696	=E'0.0625'
0006D8	40800000			697	=E'0.5'
0006DC	41100000			698	=E'1.0'
0006E0	C1200000			699	=E'-2.0'
0006E4	41900000			700	=E'9.0'
0006E8	41300000			701	=E'3.0'

APPENDIX E

Assembler Listing of GEN4

```
                                                                                          PAGE    1
  LOC  OBJECT CODE    ADDR1 ADDR2  STMT   SOURCE STATEMENT                              F08APR70   6/30/71
000000                                1 GENA     START 0
                                      2 ******************************************************************
                                      3 * SOURCE---L.E. CANNON,UGA,DEPT. OF STAT., JUN 1970               *
                                      4 * PURPOSE--TO SUPPLY THE USER WITH A FAST PROCEDURE FOR GENERATING  *
                                      5 *             BINOMIALLY DISTRIBUTED RANDOM NUMBERS.              *
                                      6 * USAGE----1. CALL BSETUP(P,IR). P IS THE PROBABILITY OF A SUCCESS. *
                                      7 *             IR IS THE RANGE OF THE VARIABLE. BSETUP SETS UP THE TABLE *
                                      8 *             FROM WHICH RANBI GENERATES A RANDOM NUMBER. BSETUP NEED BE *
                                      9 *             CALLED ONLY ONCE FOR A GIVEN SET OF PARAMETERS.      *
                                     10 *             2. X=RANBI(IOD). 'IOD' MUST BE AN ODD POSITIVE INTEGER. IT *
                                     11 *             PRIMES THE GENERATING SCHEME. X IS THE RANDOM NUMBER GENER*
                                     12 *             ATED. IT IS RETURNED IN THE REAL MODE.              *
                                     13 * EXAMPLE--CALL BSETUP(0.5,2)                                   *
                                     14 *             DO 1 I=1,100                                      *
                                     15 *             X=RANBI(55655)                                    *
                                     16 *                ...                                            *
                                     17 * METHOD---THE METHOD IS THAT OF MARSAGLIA AS DESCRIBED IN 'COMM. ACM *
                                     18 *             6',1(JAN. 1963),37.                               *
                                     19 ******************************************************************
                                     20          ENTRY BSETUP
000000 47F0 F00C           0000C     21 BSETUP   B     12(0,15)
000004 07                            22          DC    XL1'07'
000005 40C2E2C5E3E4D7                23          DC    CL7' BSETUP'
00000C 90EC D00C           0000C     24          STM   14,12,12(13)
000010 05A0                          25          BALR  10,0
000012                               26          USING *,10
000012 18ED                          27          LR    14,13
000014 41D0 A25A           0026C     28          LA    13,AREA
000018 50D0 E008           00008     29          ST    13,8(0,14)
00001C 50E0 D004           00004     30          ST    14,4(0,13)
000020 5820 1000           00000     31          L     2,0(,1)
000024 7820 2000           00000     32          LE    2,0(,2)        FPR2=P.
000028 7840 AEAE           00EC0     33          LE    4,=E'1.0'
00002C 3804                          34          LER   0,4
00002E 3B42                          35          SER   4,2          FPR4=1-P=Q.
000030 7040 A2A2           002B4     36          STE   4,Q
000034 5860 1004           00004     37          L     6,4(,1)
000038 5860 6000           00000     38          L     6,0(,6)         R6=N.
00003C 1856                          39          LR    5,6
00003E 8950 0002           00002     40          SLL   5,2            CONVERT N TO WORDS.
000042 3B66                          41          SER   6,6
000044 5060 A2A6           002B8     42          ST    6,NUM          CONVERTING TO FLOATING POINT.
000048 9246 A2A6     002B8           43          MVI   NUM,X'46'
00004C 7A60 A2A6           002B8     44          AE    6,NUM
000050 5870 AEB2           00EC4     45          L     7,=F'-1'
000054 1A67                          46          AR    6,7
000056 3C04                          47          MER   0,4            FPR0=Q**N.
000058 8667 A044           00056     48          BXH   6,7,*-2
00005C 4140 0004           00004     49          LA    4,4
000060 1B33                          50          SR    3,3
000062 3B44                          51          SER   4,4
000064 41C0 0001           00001     52          LA    12,1
000068 7900 A2B2           002C4     53          CE    0,LLIM          IF Q**N<LLIM
00006C 4740 A08E           000A0     54          BC    4,LO          BRANCH TO LO
000070 5030 A2AE           002C0     55          ST    3,XST           OTHERWISE STORE R3=0 INTO XST.
```

```
LOC    OBJECT CODE    ADDR1 ADDR2  STMT   SOURCE STATEMENT

000074 7C03 A2B6            002C8   56          STE   0,P(3)       AND STORE Q**N INTO P(0).
000078 1A34                         57          AR    3,4
00007A 7A40 AFAE            00EC0   58 L1       AF    4,=E'1.0'    FPR4=X+1.
00007E 3D04                         59          DER   0,4          FPR0=P(X)/(X+1).
000080 3C06                         60          MER   0,6          FPR0=P(X)*(N-X)/(X+1).
000082 7D00 A2A2            002B4   61          DE    0,0          FPR0=P(X)*(N-X)/((X+1)*Q).
000086 3C02                         62          MER   0,2          FPR0=P(X)*P*(N-X)/((X+1)*Q)
000088 7903 A2B2            002C4   63          CE    0,LLIM       IF FPR0=P(X+1)<LLIM
00008C 4740 A0B8            000CA   64          BC    4,CONVER     BRANCH TO CONVER
000090 7003 A2B6            002C8   65          STE   0,P(3)       OTHERWISE STORE FPR0 INTO P(I).
000094 7B60 AEAE            00EC0   66          SE    6,=E'1.0'    FPR6=N-X.
000098 8734 A068            0007A   67          BXLE  3,4,L1       CONTINUE
00009C 47F0 A0B8            000CA   68          B     CONVER
0000A0 7A40 AEAE            00EC0   69 L0       AE    4,=E'1.0'    THIS SECTION OF CODE
0000A4 3D04                         70          DER   0,4          COMPUTES SUCCESSIVE
0000A6 3C06                         71          MER   0,6          PROBABILITIES UNTIL A
0000A8 7D00 A2A2            002B4   72          DE    0,0          PROBABILITY>=LLIM IS
0000AC 3C02                         73          MER   0,2          ATTAINED, THE PROPER
0000AE 1B54                         74          SR    5,4
0000B0 7B60 AEAE            00EC0   75          SE    6,=E'1.0'
0000B4 5AC0 AEB6            00EC8   76          A     12,=F'1'
0000B8 7903 A2B2            002C4   77          CE    0,LLIM       STARTING POINT IS
0000BC 4740 A08E            000A0   78          BC    4,L0         THEN STORED
0000C0 50C0 A2AE            002C0   79          ST    12,XST       AND A BRANCH IS
0000C4 1B54                         80          SR    5,4
0000C6 47F0 A068            0007A   81          BC    15,L1        MADE TO L1 FOR FURTHER CALCULATIONS.
0000CA 1853                         82 CONVER   LR    5,3
0000CC 1B54                         83          SR    5,4
0000CE 1B33                         84          SR    3,3          R3=I.
0000D0 7823 A2B6            002C8   85 L3       LE    2,P(3)       CONVERTING TO FIXED POINT.
0000D4 7E20 A6DA            006EC   86          AU    2,7FR0
0000D8 7023 A2B6            002C8   87          STE   2,P(3)
0000DC 5893 A2B6            002C8   88          L     9,P(3)       R9=P(I).
0000E0 8990 0008            00008   89          SLL   9,8
0000E4 5093 A2B6            002C8   90          ST    9,P(3)       STORE CONVERTED P(I).
0000E8 8734 A0BE            000D0   91          BXLE  3,4,L3       CONTINUE
0000EC 1BCC                         92          SR    12,12
0000EE 1BBB                         93          SR    11,11
0000F0 1B77                         94          SR    7,7
0000F2 1B66                         95          SR    6,6
0000F4 1B33                         96          SR    3,3          R3=I.
0000F6 1B88                         97 L4       SR    8,8          CLEAR R8.
0000F8 5893 A2B6            002C8   98          L     9,P(3)       R9=P(I).
0000FC 8D80 0004            00004   99          SLDL  8,4          R8=D1.
000100 1A68                        100          AR    6,8          R6=SUM(D1).
000102 8D80 0004            00004  101          SLDL  8,4          R8=D1D2.
000106 1A78                        102          AR    7,8          R7=SUM(D1D2).
000108 8D80 0004            00004  103          SLDL  8,4          R8=D1D2D3.
00010C 1AB8                        104          AR    11,8         R11=SUM(D1D2D3).
00010E 8D80 0004            00004  105          SLDL  8,4          R8=D1D2D3D4.
000112 1AC8                        106          AR    12,8         R12=SUM(D1D2D3D4).
000114 8734 A0E4            000F6  107          BXLE  3,4,L4       CONTINUE.
000118 5060 A6B6            006C8  108          ST    6,S          STORE SUM(D1) INTO S(0).
00011C 5070 A6BA            006CC  109          ST    7,S+4        STORE SUM(D1D2) INTO S(1).
000120 50B0 A6BE            006D0  110          ST    11,S+8       STORE SUM(D1D2D3) INTO S(2)
```

```
LOC    OBJECT CODE    ADDR1 ADDR2  STMT   SOURCE STATEMENT

000124 50C0 A6C2            006D4   111          ST    12,S+12      STORE SUM(D1D2D3D4) INTO S(3).
000128 89B0 0004            00004   112          SLL   11,4         R11=SUM(D1D2D3)*16.
00012C 8970 0004            00004   113          SLL   7,4          R7=SUM(D1D2)*16.
000130 8960 0004            00004   114          SLL   6,4          R6=SUM(D1)*16.
000134 1B33                         115          SR    3,3          R3=I.
000136 1B88                         116 L5       SR    8,8          CLEAR R8.
000138 5893 A2B6            002C8   117          L     9,P(3)       R9=P(I).
00013C 8C80 0004            00004   118          SLDL  8,4          R8=D1.
000140 1B68                         119          SR    6,8          R6=SUM(D1)*16-SUM(D1).
000142 1B78                         120          SR    7,8          R7=SUM(D1D2)*16-SUM(D1).
000144 1BB8                         121          SR    11,8         R11=SUM(D1D2D3)*16-SUM(D1).
000146 1B88                         122          SR    8,8          CLEAR R8.
000148 8C80 0004            00004   123          SLDL  8,4          R8=D2.
00014C 1B78                         124          SR    7,8          R7=R7-SUM(D2).
00014E 1BB8                         125          SR    11,8         R11=R11-SUM(D2).
000150 1B88                         126          SR    8,8          CLEAR R8.
000152 8C80 0004            00004   127          SLDL  8,4          R8=D3
000156 1BB8                         128          SR    11,8         R11=R11-SUM(D3).
000158 8734 A124            00136   129          BXLE  3,4,L5       CONTINUE.
00015C 1BCC                         130          SR    12,12
00015E 5060 A6C6            006D8   131          ST    6,N          STORE R6 INTO N(3).
000162 5070 A6CA            006DC   132          ST    7,N+4        STORE R7 INTO N(1).
000166 50B0 A6CE            006E0   133          ST    11,N+8       STORE R11 INTO N(2).
00016A 1B77                         134          SR    7,7
00016C 1B00                         135          SR    0,0
00016E 4160 0001            00001   136          LA    6,1
000172 1B33                         137 L6       SR    3,3          R3=I.
000174 5820 A2AE            002C0   138          L     2,YST        LOAD R2 WITH STARTING VALUE.
000178 1B88                         139 L7       SR    8,8          CLEAR R8.
00017A 5893 A2B6            002C8   140          L     9,P(3)       R9=P(I).
00017E 8C80 0004            00004   141          SLDL  8,4          R9=D1,R8=D2,R8=D3,R8=D4.
000182 5093 A2B6            002C8   142          ST    9,P(3)       RESTORE SHIFTED P(I).
000186 5980 AE5A            00ECC   143          C     8,=F'0'
00018A 4780 A192            001A4   144          BC    8,L8         BRANCH TO L8.
00018E 1A78                         145          AR    7,8          OTHERWISE STORE R7(DISCRETE R.V.)
000190 5970 AE8E            00ED0   146          C     7,=F'2000'   CHECK THAT TABLE LENGTH IS NOT EXCEEDED.
000194 4720 A1A0            001B2   147          BC    2,L9
000198 1B76                         148          SR    7,6          THE NUMBER OF TIMES CORRESPONDING
00019A 422C A6DF            006FC   149          STC   2,A(12)      TO THE DIGIT IN R8.
00019E 87C4 A1B8            0019A   150          BXLE  12,6,*-4
0001A2 1A76                         151          AR    7,6
0001A4 1A26                         152 L8       AR    2,6          INCREMENT DISCRETE R.V.
0001A6 8734 A166            00178   153          BXLE  3,4,L7       CONTINUE.
0001AA 1A06                         154          AR    0,6          CONTINUE L6 LOOP
0001AC 1904                         155          CR    0,4          UNTIL FIRST 4 DIGITS OF
0001AE 4740 A160            00172   156          BC    4,L6         EACH P(I) HAVE BEEN SHIFTED.
0001B2 5830 A6C2            006D4   157 L9       L     3,S+12       THIS SECTION OF CODE
0001B6 5B30 A6CE            006E0   158          S     3,N+8        RESETS N(3)
0001BA 193C                         159          CR    3,12         IF TABLE LENGTH IS EXCEEDED.
0001BC 4780 A1B8            001CA   160          BC    8,OUT
0001C0 1B3C                         161          SR    3,12
0001C2 5A30 A6CE            006E0   162          A     3,N+8
0001C6 5030 A6CE            006E0   163          ST    3,N+8
0001CA                              164 OUT      DS    0H
0001CA 58D0 A25E            00270   165          L     13,AREA+4
```

```
  LOC  OBJECT CODE    ADDR1 ADDR2  STMT   SOURCE STATEMENT

0001CE 98EC D00C            0000C   166            LM    14,12,12(13)
0001D2 92FF D00C      0000C         167            MVI   12(13),X'FF'
0001D6 07FE                         168            BCR   15,14             BRANCH TO CALLING ROUTINE.
                                    169            DROP  10
                                    170            ENTRY RANBI
0001E0                              171            USING RANBI,15
0001E0 07                           172            DC    XL1'07'
0001E1 404009C105C2C9               173            DC    CL7'  RANBI'
0001E8 9013 D018            00018   174 RANBI      STM   1,3,24(13)
0001EC 1B00                         175            SER   0,0
0001EE 47F0 F070            00250   176 BRANCH     B     TEMP              BRANCH MADE ON FIRST CALL ONLY.
0001F2 5C20 F504            006E4   177            M     2,BPAT            GENERATE NEW U.
0001F6 5030 F508            006E8   178            ST    3,LA              STORE FOR NEXT CALL.
0001FA 1B22                         179            SR    2,2
0001FC 1B00                         180            SR    0,0
0001FE 8D20 0004            00004   181            SLDL  2,4               R2=D1.
000202 5920 F4E8            006C8   182            C     2,S               IF D1<S THEN
000206 4740 F05A            0023A   183            BC    4,EX              BRANCH TO EX
00020A 8D20 0004            00004   184 N1         SLDL  2,4               R2=D1D2.
00020E 5920 F4EC            006CC   185            C     2,S+4             IF D1D2>=S(1) THEN
000212 47B0 F036            00216   186            BC    11,N2             BRANCH TO N2
000216 5B20 F4F8            006D8   187            S     2,N               OTHERWISE OBTAIN PROPER TABLE ADDRESS
00021A 47F0 F05A            0023A   188            B     EX                BRANCH TO EX.
00021E 8D20 0004            00004   189 N2         SLDL  2,4               R2=D1D2D3.
000222 5920 F4F0            006D0   190            C     2,S+8             IF D1D2D3>=S(2) THEN
000226 47B0 F04A            0022A   191            BC    11,N3             BRANCH TO N3
00022A 5B20 F4FC            006DC   192            S     2,N+4             OBTAIN PROPER TABLE ADDRESS
00022E 47F0 F05A            0023A   193            B     EX                BRANCH TO EX.
000232 8D20 0004            00004   194 N3         SLDL  2,4               R2=D1D2D3D4.
000236 5920 F4F4            006D4   195            C     2,S+12            IF D1D2D3D4>=S(3) THEN
00023A 47B0 F006            001E6   196            BC    11,BRANCH         GO TO BRANCH(OBTAIN NEW U).
00023E 5B20 F500            006E0   197            S     2,N+8             OBTAIN PROPER TABLE ADDRESS.
000242 4302 F510            006F0   198 EX         IC    0,A(2)            OBTAIN PROPER DISCRETE VALUE.
000246 5000 F0DC            002BC   199            ST    0,RES             CONVERT TO FLOATING POINT.
00024A 9246 F0DC      002BC         200            MVI   RES,X'46'
00024E 7A00 F0DC            002BC   201            AE    0,RES             NORMALIZED FLT. PT. RESULT IN FPR0.
000252 9813 D018            00018   202            LM    1,3,24(13)
000256 07FE                         203            BCR   15,14             RETURN.
000258 5830 F508            006E8   204 TEMP       L     3,LA              REPLACES THE INSTRUCTION AT BRANCH.
00025C D203 F006 F070 001E6 00250   205            MVC   BRANCH,TEMP       REPLACE INSTRUCTION AT BRANCH.
000262 5830 1000            00000   206            L     3,0(,1)           LOAD PRIMER(ONLY DONE ON FIRST CALL).
000266 5830 3000            00000   207            L     3,0(,3)
00026A 5030 F508            006E8   208            ST    3,LA
00026E 47F0 F00A            001EA   209            B     BRANCH+4          GO TO INSTRUCTION FOLLOWING BRANCH.
000274C                             210 AREA       DS    18F
0002B4                              211 O          DS    1F
0002B8                              212 NUM        DS    1F
0002BC                              213 RES        DS    1F
0002C0                              214 XST        DS    1F
0002C4 3CFBA982                     215 LLIM       DC    E'0.000015'
0002C8                              216 P          DS    256F
0006C8                              217 S          DS    4F
0006D8                              218 N          DS    3F
0006E4 48C27395                     219 BPAT       DC    XL4'48C27395'
0006E8                              220 LA         DS    1F
```

```
                                                                                    PAGE    5
  LOC  OBJECT CODE    ADDR1 ADDR2  STMT   SOURCE STATEMENT                          F08APR70    6/30/71

0006EC 40000000                     221 ZERO     DC     X'40000000'
0006F0                              222 A        DS     2000XL1
                                    223          END
000FC0 41100000                     224                 =E'1.0'
000FC4 FFFFFFFF                     225                 =F'-1'
000FC8 00000001                     226                 =F'1'
000FCC 00000000                     227                 =F'0'
000FD0 000007D0                     228                 =F'2000'
```

NOT REPRODUCIBLE

APPENDIX F

Assembler Listing of GEN5

```
  LOC  OBJECT CODE    ADDR1 ADDR2  STMT   SOURCE STATEMENT                                              F08APR70   6/30/71
000000                                1 GENS     START 0
                                      2          PRINT ON,NOGEN,NODATA
                                      3 ****************************************************************************
                                      4 * SOURCE---L.E. CANNON,UGA,DEPT. OF STAT., JUN 1970                          *
                                      5 * PURPOSE--TO SUPPLY THE USER WITH A FAST PROCEDURE FOR GENERATING           *
                                      6 *              POISSON DISTRIBUTED RANDOM NUMBERS.                           *
                                      7 * USAGE----1. CALL PSETUP(LAM). LAM IS THE MEAN OF THE POISSON DIS-          *
                                      8 *              TRIBUTION TO BE GENERATED. PSETUP SETS UP THE TABLE FROM      *
                                      9 *              RANPOI GENERATES A RANDOM NUMBER. PSETUP NEED BE CALLED       *
                                     10 *              ONLY ONCE FOR A GIVEN LAM.                                    *
                                     11 *              2. X=RANPOI(IOD). 'IOD' MUST BE AN ODD INTEGER. IT PRIMES     *
                                     12 *              THE GENERATING SCHEME. X REPRESENTS THE POISSON VARIABLE      *
                                     13 *              DO 1 I=1,100                                                  *
                                     14 *              X=RANPOI(56951)                                               *
                                     15 *                 ...                                                        *
                                     16 * METHOD---THE METHOD IS THAT OF MARSAGLIA AS DESCRIBED IN 'COMM. ACM       *
                                     17 *              6',1(JAN. 1963),37.                                           *
                                     18 ****************************************************************************
                                     19          ENTRY PSETUP
000000 47F0 F00C          0000C      20 PSETUP   B     12(0,15)
000004 07                            21          DC    XL1'07'
000005 4CD7E2C5E3E4D7                22          DC    CL7'PSETUP'
00000C 90EC D00C          0000C      23          STM   14,12,12(13)
000010 05A0                          24          BALR  10,0
000012                               25          USING *,10
000012 18ED                          26          LR    14,13
000014 41D0 A232          00244      27          LA    13,AREA
000018 50D0 E008          00008      28          ST    13,8(0,14)
00001C 50E0 D004          00004      29          ST    14,4(0,13)
000020 5820 1000          00000      30          L     2,0(0,1)           LOAD ADDRESS OF LAM.
000024 7820 2000          00000      31          LE    2,0(0,2)           LOAD LAM INTO FPR2.
000028 3322                          32          LNER  2,2                FPR2=-LAM.
00002A 7020 A27A          0028C      33          STE   2,LAM
                                     34          CALL  EXP,(LAM)          COMPUTE EXP(-LAM).
000046 3022                          46          LPER  2,2                FPR2=+LAM
000048 1B55                          47          SR    5,5                INDEX.
00004A 4160 0004          00004      48          LA    6,4                INCREMENT.
00004E 4170 03FC          003FC      49          LA    7,1020             DELIMITER.
000052 58C0 AE86          00E98      50          L     12,=F'1'
000056 3B44                          51          SER   4,4
000058 7900 A286          00298      52          CE    0,LLIM             IF EXP(-LAM)<LLIM.
00005C 4740 A074          00086      53          BC    4,L0               BRANCH TO L0.
000060 7005 A28A          0029C      54          STE   0,P(5)             OTHERWISE STORE INTO P(0).
000064 5050 A282          00294      55          ST    5,XST              STORE STARTING POINT.
000068 1A56                          56          AR    5,6
00006A 7A40 AE8A          00E9C      57 L2       AE    4,=E'1'          FPR4=X.
00006E 3C02                          58          MER   0,2                FPR0=EXP(-LAM)*LAM.
000070 3D04                          59          DER   0,4                FPR0=EXP(-LAM)*LAM/X.
000072 7900 A286          00298      60          CE    0,LLIM             IF FPR0<LLIM.
000076 4740 A090          000A2      61          BC    4,CONVER           BRANCH TO CONVER.
00007A 7005 A28A          0029C      62          STE   0,P(5)             OTHERWISE STORE FPR0 INTO P(I).
00007E 8756 A058          0006A      63          BXLE  5,6,L2             CONTINUE
000082 47F0 A090          000A2      64          B     CONVER
000086 7A40 AE8A          00E9C      65 L0       AE    4,=E'1'            THIS SECTION OF CODE
00008A 3C02                          66          MER   0,2                COMPUTES SUCCESSIVE PROBILITIES
```

```
  LOC  OBJECT CODE    ADDR1 ADDR2  STMT   SOURCE STATEMENT                                          F0BAPR70  6/30/71

00008C 3D04                          67          DER   0,4          USING THE RECURSIVE RELATION UNTIL
00008E 5AC0 AE86             00E98   68          A     12,=F'1'     A PROBILITY>=LLIM IS ATTAINED.
000092 7900 A286             00298   69          CE    0,LLIM       THE PROPER STARTING POINT IS STORED.
000096 4740 A074             00086   70          BC    4,L0         AND A BRANCH IS
00009A 50C0 A282             00294   71          ST    12,XST       MADE TO L2 FOR SUB-
00009E 47F0 A058             0006A   72          BC    15,L2        SEQUENT CALCULATIONS.
0000A2 1B56                          73 CONVER   SR    5,6
0000A4 4140 0004             00004   74          LA    4,4
0000A8 1B33                          75          SR    3,3          R3=I.
0000AA 7823 A28A             0029C   76 L3       LE    2,P(3)       CONVERTING TO FIXED POINT.
0000AE 7E20 A6AE             006C0   77          AU    2,ZERO
0000B2 7023 A28A             0029C   78          STE   2,P(3)
0000B6 5893 A28A             0029C   79          L     9,P(3)       R9=P(I).
0000BA 8990 0008             00008   80          SLL   9,8
0000BE 5093 A28A             0029C   81          ST    9,P(3)       STORE CONVERTED P(I).
0000C2 8734 A098             000AA   82          BXLE  3,4,L3       CONTINUE
0000C6 1BCC                          83          SR    12,12
0000C8 1BBB                          84          SR    11,11
0000CA 1B77                          85          SR    7,7
0000CC 1B66                          86          SR    6,6
0000CE 1B33                          87          SR    3,3          R3=I.
0000D0 1B88                          88 L4       SR    8,8          CLEAR R8.
0000D2 5893 A28A             0029C   89          L     9,P(3)       R9=P(I).
0000D6 8D80 0004             00004   90          SLDL  8,4          R8=D1.
0000DA 1A68                          91          AR    6,8          R6=SUM(D1).
0000DC 8D80 0004             00004   92          SLDL  8,4          R8=D1D2.
0000E0 1A78                          93          AR    7,8          R7=SUM(D1D2).
0000E2 8D80 0004             00004   94          SLDL  8,4          R8=D1D2D3.
0000E6 1AB8                          95          AR    11,8         R11=SUM(D1D2D3).
0000E8 8D80 0004             00004   96          SLDL  8,4          R8=D1D2D3D4.
0000EC 1AC8                          97          AR    12,8         R12=SUM(D1D2D3D4).
0000EE 8734 A0BE             000D0   98          BXLE  3,4,L4       CONTINUE.
0000F2 5060 A68A             0069C   99          ST    6,S          STORE SUM(D1) INTO S(0).
0000F6 5070 A68E             006A0  100          ST    7,S+4        STORE SUM(D1D2) INTO S(1).
0000FA 50B0 A692             006A4  101          ST    11,S+8       STORE SUM(D1D2D3) INTO S(2)
0000FE 50C0 A696             006A8  102          ST    12,S+12      STORE SUM(D1D2D3D4) INTO S(3).
000102 89B0 0004             00004  103          SLL   11,4         R11=SUM(D1D2D3)*16.
000106 8970 0004             00004  104          SLL   7,4          R7=SUM(D1D2)*16.
00010A 8960 0004             00004  105          SLL   6,4          R6=SUM(D1)*16.
00010E 1B33                         106          SR    3,3          R3=I.
000110 1B88                         107 L5       SR    8,8          CLEAR R8.
000112 5893 A28A             0029C  108          L     9,P(3)       R9=P(I).
000116 8D80 0004             00004  109          SLDL  8,4          R8=D1.
00011A 1B68                         110          SR    6,8          R6=SUM(D1)*16-SUM(D1).
00011C 1B78                         111          SR    7,8          R7=SUM(D1D2)*16-SUM(D1).
00011E 1BB8                         112          SR    11,8         R11=SUM(D1D2D3)*16-SUM(D1).
000120 1B88                         113          SR    8,8          CLEAR R8.
000122 8D80 0004             00004  114          SLDL  8,4          R8=D2.
000126 1B78                         115          SR    7,8          R7=R7-SUM(D2).
000128 1BB8                         116          SR    11,8         R11=R11-SUM(D2).
00012A 1B88                         117          SR    8,8          CLEAR R8.
00012C 8D80 0004             00004  118          SLDL  8,4          R8=D3
000130 1BB8                         119          SR    11,8         R11=R11-SUM(D3).
000132 8734 ACFE             00110  120          BXLE  3,4,L5       CONTINUE.
000136 1BCC                         121          SR    12,12
```

```
  LOC  OBJECT CODE    ADDR1 ADDR2  STMT   SOURCE STATEMENT

000138 5060 A69A            006AC   122           ST    6,N            STORE R6 INTO N(0).
00013C 5070 A69E            006B0   123           ST    7,N+4          STORE R7 INTO N(1).
000140 5CB0 A6A2            006B4   124           ST    11,N+8         STORE R11 INTO N(2).
000144 1877                         125           SR    7,7
000146 1B00                         126           SR    0,0
000148 4160 0001            00001   127           LA    6,1
00014C 1B33                         128 L6        SR    3,3            R3=I.
00014E 5820 A282            00294   129           L     2,XST          LOAD R2 WITH STARTING VALUE.
000152 1B88                         130 L7        SR    8,8            CLEAR R8.
000154 5893 A28A            0029C   131           L     9,P(3)         R9=P(I).
000158 8C80 0004            00004   132           SLDL  8,4            R8=D1,R8=D2,R8=D3,R8=D4.
00015C 5093 A28A            0029C   133           ST    9,P(3)         RESTORE SHIFTED P(I).
000160 5980 AEBE            00EA0   134           C     8,=F'0'        IF R8=0.
000164 4780 A16C            0017E   135           BC    8,L9           BRANCH TO L8.
000168 1A78                         136           AR    7,8            OTHERWISE STORE R2(DISCRETE R.V.)
00016A 5970 AE92            00EA4   137           C     7,=F'2000'     CHECK THAT TABLE LENGTH IS NOT EXCEEDED.
00016E 4720 A17A            001AC   138           BC    2,L9
000172 1B76                         139           SR    7,6            THE NUMBER OF TIMES CORRESPONDING
000174 4270 A6B2            006C4   140           STC   7,A(12)        TO THE DIGIT IN R8.
000178 87C6 A162            00174   141           BXLE  12,6,*-4
00017C 1A76                         142           AR    7,6
00017E 1A26                         143 L8        AR    2,6            INCREMENT DISCRETE R.V.
000180 8734 A140            00152   144           BXLE  3,4,L7         CONTINUE.
000184 1A06                         145           AR    0,6            CONTINUE L6 LOOP
000186 1904                         146           CR    0,4            UNTIL FIRST 4 DIGITS OF
000188 4740 A13A            0014C   147           BC    4,L6           EACH P(I) HAVE BEEN SHIFTED.
00018C 5830 A696            006A8   148 L9        L     3,S+12
000190 5B30 A6A2            006B4   149           S     3,N+8          THIS SECTION OF CODE
000194 193C                         150           CR    3,12           RESETS N(3)
000196 4700 A192            001A4   151           BC    8,OUT          IF TABLE LENGTH IS EXCEEDED.
00019A 1B3C                         152           SR    3,12
00019C 5A30 A6A2            006B4   153           A     3,N+8
0001A0 5030 A6A2            006B4   154           ST    3,N+8
0001A4                              155 OUT       DS    0H
0001A4 58D0 A236            00248   156           L     13,AREA+4
0001A8 98EC D00C            0000C   157           LM    14,12,12(13)
0001AC 92FF D00C      C00C          158           MVI   12(13),X'FF'
0001B0 07FE                         159           BCR   15,14          BRANCH TO CALLING ROUTINE.
                                    160           DROP  10
                                    161           ENTRY RANPOI
0001B4                              162           USING RANPOI,15
0001B2 07                           163           DC    XL1'07'
0001B3 4009C1D5D7D6C9               164           DC    CL7' RANPOI'
0001BA 9013 D018            00018   165 RANPOI    STM   1,3,24(13)
0001BE 3B00                         166           SER   0,0
0001C0 47F0 F070            0022A   167 BRANCH    B     TEMP           BRANCH MADE ON FIRST CALL ONLY.
0001C4 5C20 F4FF            006B8   168           M     2,BPAT         GENERATE NEW U.
0001C8 5030 F502            006BC   169           ST    3,LA           STORE FOR NEXT CALL.
0001CC 1B22                         170           SR    2,2
0001CE 1B00                         171           SR    0,0
0001D0 8C20 0004            00004   172           SLDL  2,4            P2=D1.
0001D4 5920 F4E2            0069C   173           C     2,S            IF D1<S THEN
0001D8 4740 F05A            00214   174           BC    4,EX           BRANCH TO EX
0001DC 8C20 0004            00004   175 N1        SLDL  2,4            P2=D1D2.
0001E0 5920 F4E6            006A0   176           C     2,S+4          IF D1D2>=S(1) THEN
```

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  LOC  OBJECT CODE    ADDR1 ADDR2  STMT   SOURCE STATEMENT                                          F08APR70   6/30/71

0001E4 4780 F036            001FC   177          BC    11,N2          BRANCH TO N2
0001F8 5820 F4F2            006AC   178          S     2,N            OTHERWISE OBTAIN PROPER TABLE ADDRESS
0001FC 47F0 F05A            00214   179          B     EX             BRANCH TO EX.
0001F0 8D20 0004            00004   180 N2       SLDL  2,4            R2=D1D2D3.
0001F4 5920 F4EA            006A4   181          C     2,S+8          IF D1D2D3>=S(2) THEN
0001F8 47B0 F04A            00204   182          BC    11,N3          BRANCH TO N3
0001FC 5B20 F4F6            006B0   183          S     2,N+4          OBTAIN PROPER TABLE ADDRESS
000200 47F0 F05A            00214   184          B     EX             BRANCH TO EX.
000204 8C20 0004            00004   185 N3       SLDL  2,4            R2=D1D2D3D4.
000208 5920 F4EE            006A8   186          C     2,S+12         IF D1D2D3D4>=S(3) THEN
00020C 47B0 F006            001C0   187          BC    11,BRANCH      GO TO BRANCH(OBTAIN NEW U).
000210 5B20 F4FA            006B4   188          S     2,N+8          OBTAIN PROPER TABLE ADDRESS.
000214 4302 F50A            006C4   189 EX       IC    0,A(2)         OBTAIN PROPER DISCRETE VALUE.
000218 5000 F006            00290   190          ST    0,RES          CONVERT TO FLOATING POINT.
00021C 9246 F006      00290         191          MVI   RES,X'46'
000220 7A00 F006            00290   192          AE    0,RES          NORMALIZED FLT. PT. RESULT IN FPR0.
000224 9813 0018            00018   193          LM    1,3,24(13)
000228 07FE                         194          BCR   15,14          RETURN.
00022A 5830 F502            006BC   195 TEMP     L     3,LA           REPLACES THE INSTRUCTION AT BRANCH.
00022E D203 F006 F070 001C0 0022A   196          MVC   BRANCH,TEMP    REPLACE INSTRUCTION AT BRANCH.
000234 5830 1000            00000   197          L     3,0(,1)        LOAD PRIMER(ONLY DONE ON FIRST CALL).
000238 5830 3000            00000   198          L     3,0(,3)
00023C 5030 F502            006BC   199          ST    3,LA
000240 47F0 F00A            001C4   200          B     BRANCH+4       GO TO INSTRUCTION FOLLOWING BRANCH.
000244                              201 AREA     DS    18F
00028C                              202 LAM      DS    1F
000290                              203 RES      DS    1F
000294                              204 XST      DS    1F
000298 3CFBA882                     205 LLIM     DC    F'0.000015'
00029C                              206 P        DS    256F
00069C                              207 S        DS    4F
0006AC                              208 N        DS    3F
0006B8 48C27395                     209 BPAT     DC    XL4'48C27395'
0006BC                              210 LA       DS    1F
0006C0 4000C000                     211 ZERO     DC    X'40000000'
0006C4                              212 A        DS    2000XL1
                                    213          END
000F98 00000001                     214                =F'1'
000F9C 41100000                     215                =E'1'
000FA0 00000000                     216                =F'0'
000FA4 000007D0                     217                =F'2000'
```

APPENDIX G

Assembler Listing of GEN6

```
  LOC  OBJECT CODE    ADDR1 ADDR2  STMT   SOURCE STATEMENT

000000                               1 GEN6     START 0
                                     2 ***********************************************************************
                                     3 * SOURCE---L.F. CANNON,UGA,DEPT. OF STAT., JUN 1970                  *
                                     4 * PURPOSE--TO SUPPLY THE USER WITH A FAST PROCEDURE FOR GENERATING NE-*
                                     5 *          GATIVE BINOMIAL DISTRIBUTED RANDOM NUMBERS.               *
                                     6 * USAGE----1. CALL NBSETU(P,IR). P=PROBABILITY OF A SUCCESS, IR=NUM- *
                                     7 *          BER OF SUCCESSES. NBSETU SETS UP THE TABLE FROM WHICH RANE-*
                                     8 *          BI GENERATES A RANDOM NUMBER. NBSETU NEED BE CALLED ONLY   *
                                     9 *          ONCE FOR A GIVEN P AND IR.                                 *
                                    10 *          2. X=RANEBI(IOD). 'IOD' MUST BE AN ODD INTEGER, IT PRIMES  *
                                    11 *          THE GENERATING SCHEME. X REPRESENTS THE NUMBER OF FAILURES *
                                    12 *          BEFORE THE IR'TH SUCCESS.                                  *
                                    13 * EXAMPLE--CALL NBSETU(0.5,10)                                        *
                                    14 *          DO 1 I=1,100                                               *
                                    15 *          X=RANEBI(56695)                                            *
                                    16 *          ...                                                        *
                                    17 * METHOD---THE METHOD IS THAT OF MARSAGLIA AS DESCRIBED IN 'COMM. ACM *
                                    18 *          6',1(JAN. 1963),37.                                        *
                                    19 ***********************************************************************
                                    20          ENTRY NBSETU
000000 47F0 F00C            0000C   21 NBSETU   B     12(0,15)
000004 07                           22          DC    XL1'07'
000005 4005C2C5E3E4                 23          DC    CL7' NBSETU'
00000C 90EC D00C            000CC   24          STM   14,12,12(13)
000010 05A0                         25          BALR  10,0
000012                              26          USING *,10
000012 18ED                         27          LR    14,13
000014 41D0 A24E            00260   28          LA    13,AREA
000018 50D0 E008            00008   29          ST    13,8(0,14)
00001C 50E0 D004            00004   30          ST    14,4(0,13)
000020 5840 1000            00000   31          L     4,0(,1)            LOAD ADDRESS OF P.
000024 7840 4000            00000   32          LE    4,0(,4)            LOAD P INTO FPR4.
000028 7820 AE9E            00EB0   33          LE    2,=E'1'
00002C 3B24                         34          SER   2,4                CALCULATE Q=1-P.
00002E 5860 1004            00004   35          L     6,4(,1)            LOAD ADDRESS OF R.
000032 5860 6000            00000   36          L     6,0(,6)            LOAD R.
000036 5060 A29A            002AC   37          ST    6,SUC              CONVERT R TO FLOATING POINT.
00003A 9246 A29A      002AC         38          MVI   SUC,X'46'
00003E 7800 AE9E            00EB0   39          LE    0,=E'1'
000042 3B66                         40          SER   6,6
000044 7A60 A29A            002AC   41          AE    6,SUC              FPR6=R (NORMALIZED FLT. PT.).
000048 4780 A046            00058   42          BC    8,SKIP             IF R=0 BRANCH TO SKIP
00004C 5870 AEA2            00EB4   43          L     7,=F'-1'
000050 1A67                         44          AR    6,7
000052 3C04                         45          MER   0,4                CALCULATE P**R.
000054 8667 A040            00052   46          BXH   6,7,*-2
000058 1B55                         47 SKIP     SR    5,5
00005A 4160 0004            00004   48          LA    6,4
00005E 4170 03FC            003FC   49          LA    7,1020
000062 58C0 AEA6            00EB8   50          L     12,=F'1'
000066 3B44                         51          SER   4,4
000068 7900 A2A2            002B4   52          CE    0,LLIM             IF P**R<LLIM.
00006C 4740 A08A            0009C   53          BC    4,LO               BRANCH TO LO.
000070 7005 A2A6            002B8   54          STE   0,P(5)             OTHERWISE STORE P**R INTO P(0).
000074 505D A29E            002B0   55          ST    5,XST              STORE R5=0 INTO XST.
```

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LOC    OBJECT CODE    ADDR1 ADDR2  STMT   SOURCE STATEMENT

000078 1A56                         56          AR    5,6
00007A 7A40 AE9E            00EA0   57 L2       AE    4,=E'1'        FPR4=X+1.
00007E 3C06                         58          MER   0,6            FPRO=P(X)*(X+R).
000080 3D04                         59          DER   0,4            FPRO=P(X)*(X+R)/(X+1).
000082 3C02                         60          MER   0,2            FPRO=P(X+1)=P(X)*(X+R)*Q/(X+1).
000084 7900 A2A2            002B4   61          CE    0,LLIM         IF P(X+1)<LLIM,
000088 4740 A0AC            000BE   62          BC    4,CONVER       BRANCH TO CONVER.
00008C 7005 A2A6            002B8   63          STE   0,P(5)         OTHERWISE STORE FPRO=P(X+1) INTO P(R5).
000090 7A60 AE9E            00EB0   64          AE    6,=E'1'        INCREMENT X+R.
000094 8756 A068            0007A   65          BXLE  5,6,L2         CONTINUE.
000098 47F0 A0AC            000BE   66          B     CONVER
00009C 7A40 AE9E            00EB0   67 L0       AE    4,=E'1'        THIS SECTION OF CODE
0000A0 3C06                         68          MER   0,6            CALCULATES SUCCESSIVE PROBA-
0000A2 3D04                         69          DER   0,4            BILITIES, USING THE RECURSIVE
0000A4 3C02                         70          MER   0,2            RELATION, UNTIL A PROBABILITY>=LLIM
0000A6 5AC0 AEA6            00EB8   71          A     12,=F'1'       IS OBTAINED.
0000AA 7A60 AE9E            00EB0   72          AE    6,=E'1'        THE CORRESPONDING X-VALUE
0000AE 7900 A2A2            002B4   73          CE    0,LLIM         IS STORED INTO XST
0000B2 4740 A08A            0009C   74          BC    4,L0           AND A BRANCH
0000B6 50C0 A29E            002B0   75          ST    12,XST         IS MADE TO L2
0000BA 47F0 A068            0007A   76          BC    15,L2          FOR FUTHER CALCULATIONS.
0000BE 1B56                         77 CONVER   SR    5,6
0000C0 4140 0004            00004   78          LA    4,4
0000C4 1B33                         79          SR    3,3            R3=I.
0000C6 7823 A2A6            002B8   80 L3       LE    2,P(3)         CONVERTING TO FIXED POINT.
0000CA 7E23 A6CA            006DC   81          AU    2,ZERO
0000CE 7023 A2A6            002B8   82          STE   2,P(3)
0000D2 5893 A2A6            002B8   83          L     9,P(3)
0000D6 8990 0008            00008   84          SLL   9,8
0000DA 5093 A2A6            002B8   85          ST    9,P(3)
0000DE 8734 A0B4            000C6   86          BXLE  3,4,L3         CONTINUE
0000E2 1BCC                         87          SR    12,12
0000E4 1BBB                         88          SR    11,11
0000E6 1B77                         89          SR    7,7
0000E8 1B66                         90          SR    6,6
0000EA 1B33                         91          SR    3,3            R3=I.
0000EC 1B88                         92 L4       SR    8,8            CLEAR R8.
0000EE 5893 A2A6            002B8   93          L     9,P(3)         R9=P(I).
0000F2 8D80 0004            00004   94          SLDL  8,4            R8=D1.
0000F6 1A68                         95          AR    6,8            R6=SUM(D1).
0000F8 8D80 0004            00004   96          SLDL  8,4            R8=D1D2.
0000FC 1A78                         97          AR    7,8            R7=SUM(D1D2).
0000FE 8D80 0004            00004   98          SLDL  8,4            R8=D1D2D3.
000102 1AB8                         99          AR    11,8           R11=SUM(D1D2D3).
000104 8D80 0004            00004   100         SLDL  8,4            R8=D1D2D3D4.
000108 1AC8                         101         AR    12,8           R12=SUM(D1D2D3D4).
00010A 8734 A0DA            000EC   102         BXLE  3,4,L4         CONTINUE.
00010E 5060 A6A6            006B8   103         ST    6,S            STORE SUM(D1) INTO S(0).
000112 5070 A6AA            006BC   104         ST    7,S+4          STORE SUM(D1D2) INTO S(1).
000116 50B0 A6AE            006C0   105         ST    11,S+8         STORE SUM(D1D2D3) INTO S(2)
00011A 50C0 A6B2            006C4   106         ST    12,S+12        STORE SUM(D1D2D3D4) INTO S(3).
00011E 89B0 0004            00004   107         SLL   11,4           R11=SUM(D1D2D3)*16.
000122 8970 0004            00004   108         SLL   7,4            R7=SUM(D1D2)*16.
000126 8960 0004            00004   109         SLL   6,4            R6=SUM(D1)*16.
00012A 1B33                         110         SR    3,3            R3=I.
```

```
  LOC  OBJECT CODE    ADDR1 ADDR2  STMT   SOURCE STATEMENT

00012C 1888                         111 L5       SR    8,8             CLEAR R8.
00012E 5890 A2A6            002B8   112          L     9,P(3)          R9=P(1).
000132 8C80 0004            00004   113          SLDL  8,4             R8=D1.
000136 1868                         114          SR    6,8             R6=SUM(D1)*16-SUM(D1).
000138 1878                         115          SR    7,8             R7=SUM(D1D2)*16-SUM(D1).
00013A 18B8                         116          SR    11,8            R11=SUM(D1D2D3)*16-SUM(D1).
00013C 1888                         117          SR    8,8             CLEAR R8.
00013E 8C80 0004            00004   118          SLDL  8,4             R8=D2.
000142 1878                         119          SR    7,8             R7=R7-SUM(D2).
000144 18B8                         120          SR    11,8            R11=R11-SUM(D2).
000146 1888                         121          SR    8,8             CLEAR R8.
000148 8C80 0004            00004   122          SLDL  8,4             R8=D3
00014C 18B8                         123          SR    11,8            R11=R11-SUM(D3).
00014E 8734 A11A            0012C   124          BXLE  3,4,L5          CONTINUE.
000152 1BCC                         125          SR    12,12
000154 5060 A6B6            006C8   126          ST    6,N             STORE R6 INTO N(0).
000158 5070 A6BA            006CC   127          ST    7,N+4           STORE R7 INTO N(1).
00015C 50B0 A6BE            006D0   128          ST    11,N+8          STORE R11 INTO N(2).
000160 1877                         129          SR    7,7
000162 1800                         130          SR    0,0
000164 4160 0001            00001   131          LA    6,1
000168 1833                         132 L6       SR    3,3             R3=I.
00016A 5820 A29E            002B0   133          L     2,XST           LOAD R2 WITH STARTING VALUE.
00016E 1888                         134 L7       SR    8,8             CLEAR R8.
000170 5890 A2A6            002B8   135          L     9,P(3)          R9=P(1).
000174 8C80 0004            00004   136          SLDL  8,4             R8=D1,R8=D2,R8=D3,R8=D4.
000178 5090 A2A6            002B8   137          ST    9,P(3)          RESTORE SHIFTED P(1).
00017C 5580 AEAA            00EBC   138          C     8,=F'0'         IF R8=0.
000180 4780 A188            0019A   139          BC    8,L8            BRANCH TO L8.
000184 1A78                         140          AR    7,8             OTHERWISE STORE R2(DISCRETE R.V.)
000186 5970 AFAE            00FC0   141          C     7,=F'2000'      CHECK THAT TABLE LENGTH IS NOT EXCEEDED.
00018A 4720 A196            001A8   142          BC    2,L9
00018E 1B76                         143          SR    7,6             THE NUMBER OF TIMES CORRESPONDING
000190 4220 A6CE            006E0   144          STC   2,A(12)         TO THE DIGIT IN R8.
000194 87C6 A17E            00190   145          BXLE  12,6,*-4
000198 1A76                         146          AR    7,6
00019A 1A26                         147 L8       AR    2,6             INCREMENT DISCRETE R.V.
00019C 8734 A15C            0016E   148          BXLE  3,4,L7          CONTINUE.
0001A0 1A06                         149          AR    0,6             CONTINUE L6 LOOP
0001A2 1904                         150          CR    0,4             UNTIL FIRST 4 DIGITS OF
0001A4 4740 A156            00168   151          BC    4,L6            EACH P(I) HAVE BEEN SHIFTED.
0001A8 5830 A6B2            006C4   152 L9       L     3,S+12
0001AC 5B30 A6BE            006D0   153          S     3,N+8           THIS SECTION OF CODE
0001B0 193C                         154          CR    3,12            RESETS N(3)
0001B2 4780 A1AE            001C0   155          BC    8,OUT           IF TABLE LENGTH IS EXCEEDED.
0001B6 1B3C                         156          SR    3,12
0001B8 5A30 A6BE            006D0   157          A     3,N+8
0001BC 5030 A6BE            006D0   158          ST    3,N+8
0001C0                              159 OUT      DS    0H
0001C0 58D0 A252            00264   160          L     13,AREA+4
0001C4 98EC D00C            0000C   161          LM    14,12,12(13)
0001C8 92FF D00C      0000C         162          MVI   12(13),X'FF'
0001CC 07FE                         163          BCR   15,14           BRANCH TO CALLING ROUTINE.
                                    164          DROP  10
                                    165          ENTRY RANEBI
```

```
  LOC  OBJECT CODE    ADDR1 ADDR2  STMT   SOURCE STATEMENT                                        FOBAPRTO   6/30/71

0001D6                             166           USING RANEBI,15
0001CE 07                          167           DC    XL1'07'
0001CF 40D9C1D5C5C2C9              168           DC    CL7' RANEBI'
0001D5 9013 D018            00018  169 RANEBI    STM   1,3,24(13)
0001DA 3800                        170           SER   0,0
0001DC 47F0 FC70            00246  171 BRANCH    B     TEMP          BRANCH MADE ON FIRST CALL ONLY.
0001E0 5C20 F4FE            00604  172           M     2,BPAT        GENERATE NEW U.
0001E4 5C30 F507            00608  173           ST    3,LA          STORE FOR NEXT CALL.
0001E8 1B22                        174           SR    2,2
0001EA 1B00                        175           SR    0,0
0001EC 8D20 0004            00004  176           SLDL  2,4           R2=D1.
0001F0 5920 F4E2            005E8  177           C     2,S           IF D1<S THEN
0001F4 4740 F05A            00230  178           BC    4,EX          BRANCH TO EX
0001F8 8C20 0004            00004  179 N1        SLDL  2,4           R2=D1D2.
0001FC 5920 F4F6            006AC  180           C     2,S+4         IF D1D2>=S(1) THEN
000200 47B0 F036            0020C  181           BC    11,N2         BRANCH TO N2
000204 5B20 F4F2            005C8  182           S     2,N           OTHERWISE OBTAIN PROPER TABLE ADDRESS
000208 47F0 F05A            00230  183           B     EX            BRANCH TO EX.
00020C 8C20 0004            00004  184 N2        SLDL  2,4           R2=D1D2D3.
000210 5920 F4FA            006C0  185           C     2,S+8         IF D1D2D3>=S(2) THEN
000214 47B0 F04A            00220  186           BC    11,N3         BRANCH TO N3
000218 5B20 F4F6            006CC  187           S     2,N+4         OBTAIN PROPER TABLE ADDRESS
00021C 47F0 F05A            00230  188           B     EX            BRANCH TO EX.
000220 8D20 0004            00004  189 N3        SLDL  2,4           R2=D1D2D3D4.
000224 5920 F4FE            006C4  190           C     2,S+12        IF D1D2D3D4>=S(3) THEN
000228 47B0 FC06            001DC  191           BC    11,BRANCH     GO TO BRANCH(OBTAIN NEW U).
00022C 5B20 F4FA            00600  192           S     2,N+8         OBTAIN PROPER TABLE ADDRESS.
000230 4302 F50A            006E0  193 EX        IC    0,A(2)        OBTAIN PROPER DISCRETE VALUE.
000234 5000 F002            002A8  194           ST    0,RES         CONVERT TO FLOATING POINT.
000238 9246 F002      C02A8        195           MVI   RES,X'46'
00023C 7A00 F002            002A8  196           AE    0,RES         NORMALIZED FLT. PT. RESULT IN FPR0.
000240 9813 D018            00018  197           LM    1,3,24(13)
000244 07FE                        198           BCR   15,14         RETURN.
000246 5830 F502            006D8  199 TEMP      L     3,LA          REPLACES THE INSTRUCTION AT BRANCH.
00024A D203 F006 F070 C01DC 00246  200           MVC   BRANCH,TEMP   REPLACE INSTRUCTION AT BRANCH.
000250 5831 1000            00000  201           L     3,0(,1)       LOAD PRIMER(ONLY DONE ON FIRST CALL).
000254 5833 3000            00000  202           L     3,0(,3)
000258 5030 F502            006D8  203           ST    3,LA
00025C 47F0 FC0A            001E0  204           B     BRANCH+4      GO TO INSTRUCTION FOLLOWING BRANCH.
000260                             205 AREA      DS    18F
0002A8                             206 RES       DS    1F
0002AC                             207 SUC       DS    1F
0002B0                             208 XST       DS    1F
0002B4 3CFBA882                    209 LLIM      DC    E'0.000015'
0002B8                             210 P         DS    256F
0006B8                             211 S         DS    4F
0006C8                             212 N         DS    3F
0006D4 48C27395                    213 BPAT      DC    XL4'48C27395'
0006D8                             214 LA        DS    1F
0006DC 40000000                    215 ZERO      DC    X'40000000'
0006E0                             216 A         DS    2000XL1
                                   217           END
000EB0 41100000                    218                 =E'1'
000EB4 FFFFFFFF                    219                 =F'-1'
000EB8 00000001                    220                 =F'1'
```

```
  LOC  OBJECT CODE    ADDR1 ADDR2  STMT   SOURCE STATEMENT

000EBC 00000000                    221              =F'0'
000EC0 000007D0                    222              =F'2000'
```

APPENDIX H

Assembler Listing Of GEN7

```
LOC    OBJECT CODE     ADDR1 ADDR2  STMT   SOURCE STATEMENT                                           FURAPRTO   6/30/71

000000                                 1 GEN7     START 0
                                       2 ********************************************************************
                                       3 * SOURCE---L.E. CANNON, UGA, DEPT. OF STAT., JUN 1970.             *
                                       4 * PURPOSE--TO SUPPLY THE USER WITH A FAST PROCEDURE FOR GENERATING DIS*
                                       5 *          CRETE RANDOM VARIABLES WITH A SPECIFIED PROBABILITY.     *
                                       6 * USAGE----1. CALL DSETUP(P,IX,N). 'P' AND 'IX' ARE VECTORS OF PROBA- *
                                       7 *          BILITIES AND THE CORRESPONDING DISCRETE VARIABLES ( MUST   *
                                       8 *          BE INTEGERS<256). 'N' IS THE NUMBER OF ELEMENTS IN THE     *
                                       9 *          VECTORS. DSETUP SETS UP THE TABLE FROM WHICH RANDIS GENER- *
                                      10 *          ATES A RANDOM NUMBER. DSETUP NEED BE CALLED ONLY ONCE FOR  *
                                      11 *          A GIVEN 'P','IX', AND 'N'.                                  *
                                      12 *          2. X=RANDIS(IOD). 'IOD' MUST BE AN ODD INTEGER, IT PRIMES  *
                                      13 *          THE GENERATING SCHEME. X REPRESENTS THE DESIRED RANDOM     *
                                      14 *          VARIABLE, RETURNED IN FLOATING POINT.                       *
                                      15 * METHOD---THE METHOD IS THAT OF MARSAGLIA AS DESCRIBED IN 'COMM. ACM *
                                      16 *          6',1(JAN 1963),37.                                          *
                                      17 ********************************************************************
                                      18          ENTRY TSETUP
000000 47F0 F00C            0000C     19 TSETUP   B     12(0,15)
000004 07                             20          DC    XL1'07'
000005 40E3E2C5E3E4D7                 21          DC    CL7' TSETUP'
00000C 90EC D00C            0000C     22          STM   14,12,12(13)
000010 05A0                           23          BALR  10,0
000012                                24          USING *,10
000012 18ED                           25          LR    14,13
000014 41D0 A1BE            00100     26          LA    13,AREA
000018 50D0 E008            00008     27          ST    13,8(0,14)
00001C 50E0 D004            00004     28          ST    14,4(0,13)
000020 5850 1008            00008     29          L     5,8(,1)
000024 5850 5000            00000     30          L     5,0(,5)        LOAD N INTO R5
000028 8950 0002            00002     31          SLL   5,2            CONVERT N TO WORDS (1 WORD= 4 BYTES).
00002C 58B0 1000            00000     32          L     11,0(,1)       LOAD ADDRESS OF P VECTOR INTO R11.
000030 4140 0004            00004     33          LA    4,4
000034 1B54                           34          SR    5,4
000036 1B33                           35          SR    3,3            R3=I.
000038 7823 B000            00000     36 L3       LE    2,0(3,11)      LOAD LOAD FPR2 WITH P(I) FROM PARM. LIST.
00003C 7E20 A62E            00640     37          AU    2,ZERO         CONVERTING TO FIX POINT.
000040 7023 A20A            0021C     38          STE   2,P(3)
000044 5893 A20A            0021C     39          L     9,P(3)
000048 8990 0008            00008     40          SLL   9,8
00004C 5093 A20A            0021C     41          ST    9,P(3)
000050 8734 A026            00038     42          BXLE  3,4,L3         CONTINUE
000054 1BCC                           43          SR    12,12
000056 1BBB                           44          SR    11,11
000058 1B77                           45          SR    7,7
00005A 1B66                           46          SR    6,6
00005C 1B33                           47          SR    3,3            R3=I.
00005E 1B88                           48 L4       SR    8,8            CLEAR R8.
000060 5893 A20A            0021C     49          L     9,P(3)         R9=P(I).
000064 8D80 0004            00004     50          SLDL  8,4            R8=D1.
000068 1A68                           51          AR    6,8            R6=SUM(D1).
00006A 8D80 0004            00004     52          SLDL  8,4            R8=D1D2.
00006E 1A78                           53          AR    7,8            R7=SUM(D1D2).
000070 8D80 0004            00004     54          SLDL  8,4            R8=D1D2D3.
000074 1AB8                           55          AR    11,8           R11=SUM(D1D2D3).
```

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  LOC  OBJECT CODE    ADDR1 ADDR2  STMT   SOURCE STATEMENT

000074 8D80 0004            00004    56          SLDL  8,4           R8=D1D2D3D4.
00007A 1AC9                          57          AR    12,8          R12=SUM(D1D2D3D4).
00007C 8734 A04C            0005E    58          BXLE  3,4,L4        CONTINUE.
000080 5060 A60A            0061C    59          ST    6,S           STORE SUM(D1) INTO S(0).
000084 5070 A60E            00620    60          ST    7,S+4         STORE SUM(D1D2) INTO S(1).
000088 50B0 A612            00624    61          ST    11,S+8        STORE SUM(D1D2D3) INTO S(2)
00008C 50C0 A616            00628    62          ST    12,S+12       STORE SUM(D1D2D3D4) INTO S(3).
000090 89B0 0004            00004    63          SLL   11,4          R11=SUM(D1D2D3)*16.
000094 8970 0004            00004    64          SLL   7,4           R7=SUM(D1D2)*16.
000098 8960 0004            00004    65          SLL   6,4           R6=SUM(D1)*16.
00009C 1B33                          66          SR    3,3           R3=I.
00009E 1B88                          67 L5       SR    8,8           CLEAR R8.
0000A0 5893 A20A            0021C    68          L     9,P(3)        R9=P(I).
0000A4 8D80 0004            00004    69          SLDL  8,4           R8=D1.
0000A8 1B68                          70          SR    6,8           R6=SUM(D1)*16-SUM(D1).
0000AA 1B78                          71          SR    7,8           R7=SUM(D1D2)*16-SUM(D1).
0000AC 1BB8                          72          SR    11,8          R11=SUM(D1D2D3)*16-SUM(D1).
0000AE 1889                          73          SR    8,9           CLEAR R9.
0000B0 8D80 0004            00004    74          SLDL  8,4           R8=D2.
0000B4 1B78                          75          SR    7,8           R7=R7-SUM(D2).
0000B6 1BB8                          76          SR    11,8          R11=R11-SUM(D2).
0000B8 1889                          77          SR    8,8           CLEAR R9.
0000BA 8D80 0004            00004    78          SLDL  8,4           R8=D3
0000BE 1BB8                          79          SR    11,8          R11=R11-SUM(D3).
0000C0 8734 A09C            0009E    80          BXLE  3,4,L5        CONTINUE.
0000C4 1BCC                          81          SR    12,12
0000C6 5060 A61A            0062C    82          ST    6,N           STORE R6 INTO N(0).
0000CA 5070 A61E            00630    83          ST    7,N+4         STORE R7 INTO N(1).
0000CE 50B0 A622            00634    84          ST    11,N+8        STORE R11 INTO N(2).
0000D2 1B77                          85          SR    7,7
0000D4 1B00                          86          SR    0,0
0000D6 5860 AE06            00E18    87          L     6,=F'1'
0000DA 58B0 1004            00004    88          L     11,4(,1)      LOAD R11 WITH ADDRESS OF X VECTOR.
0000DE 1B33                          89 L6       SR    3,3           R3=I.
0000E0 1B88                          90 L7       SR    8,8           CLEAR R8.
0000E2 5893 A20A            0021C    91          L     9,P(3)        R9=P(I).
0000E6 8D80 0004            00004    92          SLDL  8,4           R8=D1,R8=D2,R8=D3,R8=D4.
0000EA 5093 A20A            0021C    93          ST    9,P(3)        RESTORE SHIFTED P(I).
0000EE 5580 AE0A            00E1C    94          C     8,=F'0'       IF R8=0.
0000F2 4780 A0FE            00110    95          BC    8,L8          BRANCH TO L8.
0000F6 1A78                          96          AR    7,8           OTHERWISE STORE R2(DISCRETE R.V.)
0000F8 5570 AE0E            00E20    97          C     7,=F'2000'    CHECK THAT TABLE LENGTH IS NOT EXCEEDED.
0000FC 4720 A10A            0011C    98          BC    2,L9
000100 5823 B000            00000    99          L     2,0(3,11)     LOAD R2 WITH DISCRETE VALUE.
000104 1B76                         100          SR    7,6           THE NUMBER OF TIMES CORRESPONDING
000106 422C A632            00644   101          STC   2,A(12)       TO THE DIGIT IN R8.
00010A 87C6 A0F4            00106   102          BXLE  12,6,*-4
00010E 1A76                         103          AR    7,6
000110 8734 A0CE            000E0   104 L8       BXLE  3,4,L7        CONTINUE.
000114 1A06                         105          AR    0,6           CONTINUE L6 LOOP
000116 1904                         106          CR    0,4           UNTIL FIRST 4 DIGITS OF
000118 4740 A0CC            000DE   107          BC    4,L6          EACH P(I) HAVE BEEN SHIFTED.
00011C 5830 A616            00628   108 L9       L     3,S+12
000120 5B30 A622            00634   109          S     3,N+8         THIS SECTION OF CODE
000124 193C                         110          CR    3,12          RESETS N(3)
```

```
  LOC  OBJECT CODE    ADDR1 ADDR2  STMT   SOURCE STATEMENT

000126 4780 A122            00134   111          BC    4,OUT          IF TABLE LENGTH IS EXCEEDED.
00012A 183C                         112          SR    3,12
00012C 5A30 A622            00634   113          A     3,N+8
000130 5030 A622            00634   114          ST    3,N+8
000134                              115 OUT      DS    0H
000134 5800 A1C2            001D4   116          L     13,AREA+4
000138 98EC D00C            0000C   117          LM    14,12,12(13)
00013C 07FE                         118          BCR   15,14          BRANCH TO CALLING ROUTINE.
                                    119          DROP  10
                                    120          ENTRY RANDIS
000146                              121          USING RANDIS,15
00013E C7                           122          DC    XL1'07'
00013F 40D9C1D5C4C9E2               123          DC    CL7' RANDIS'
000146 9003 D014            00014   124 RANDIS   STM   0,3,20(13)
00014A 3300                         125          SER   0,0
00014C 47F0 F070            00186   126 BRANCH   B     TEMP           BRANCH MADE ON FIRST CALL ONLY.
000150 5C20 F4F2            00638   127          M     2,BPAT         GENERATE NEW U.
000154 5030 F4F6            0063C   128          ST    3,LA           STORE FOR NEXT CALL.
000158 1822                         129          SR    2,2
00015A 1800                         130          SR    0,0
00015C 8D20 0004            00004   131          SLDL  2,4            R2=D1.
000160 5920 F4D6            0061C   132          C     2,S            IF D1<S THEN
000164 4740 F05A            001A0   133          BC    4,EX           BRANCH TO EX
000168 8D20 0004            00004   134 N1       SLDL  2,4            R2=D1D2.
00016C 5920 F4DA            00620   135          C     2,S+4          IF D1D2>=S(1) THEN
000170 47B0 F036            0017C   136          BC    11,N2          BRANCH TO N2
000174 5B20 F4E6            0062C   137          S     2,N            OTHERWISE OBTAIN PROPER TABLE ADDRESS
000178 47F0 F05A            001A0   138          B     EX             BRANCH TO EX.
00017C 8D20 0004            00004   139 N2       SLDL  2,4            R2=D1D2D3.
000180 5920 F4DE            00624   140          C     2,S+8          IF D1D2D3>=S(2) THEN
000184 47B0 F04A            00190   141          BC    11,N3          BRANCH TO N3
000188 5B20 F4EA            00630   142          S     2,N+4          OBTAIN PROPER TABLE ADDRESS
00018C 47F0 F05A            001A0   143          B     EX             BRANCH TO EX.
000190 8D20 0004            00004   144 N3       SLDL  2,4            R2=D1D2D3D4.
000194 5920 F4E2            00628   145          C     2,S+12         IF D1D2D3D4>=S(3) THEN
000198 47B0 F006            0014C   146          BC    11,BRANCH      GO TO BRANCH(OBTAIN NEW U).
00019C 5B20 F4EE            00634   147          S     2,N+8          OBTAIN PROPER TABLE ADDRESS.
0001A0 4302 F4FE            00644   148 EX       IC    0,A(2)         OBTAIN PROPER DISCRETE VALUE.
0001A4 5000 F0D2            00218   149          ST    0,RES          CONVERT TO FLOATING POINT.
0001A8 9246 F0D2      00218         150          MVI   RES,X'46'
0001AC 7A00 F0D2            00218   151          AE    0,RES          NORMALIZED FLT. PT. RESULT IN FPR0.
0001B0 9803 D014            00014   152          LM    0,3,20(13)
0001B4 07FE                         153          BCR   15,14          RETURN.
0001B6 5830 F4F6            0063C   154 TEMP     L     3,LA           REPLACES THE INSTRUCTION AT BRANCH.
0001BA D203 F006 F070 0014C 00186   155          MVC   BRANCH,TEMP    REPLACE INSTRUCTION AT BRANCH.
0001C0 5830 1000            00000   156          L     3,0(,1)        LOAD PRIMER(ONLY DONE ON FIRST CALL).
0001C4 5830 3000            00000   157          L     3,0(,3)
0001C8 5030 F4F6            0063C   158          ST    3,LA
0001CC 47F0 F00A            00150   159          B     BRANCH+4       GO TO INSTRUCTION FOLLOWING BRANCH.
0001D0                              160 AREA     DS    19F
000218                              161 RES      DS    1F
00021C                              162 P        DS    256F
00061C                              163 S        DS    4F
00062C                              164 N        DS    3F
000638 48C27395                     165 BPAT     DC    XL4'48C27395'
```

```
  LOC  OBJECT CODE    ADDR1 ADDR2  STMT   SOURCE STATEMENT
00063C                              166 LA       DS    1F
000640 40000000                     167 ZERO     DC    X'40000000'
000644                              168 A        DS    2000XL1
                                    169          END
000E18 00000001                     170                =F'1'
000E1C 00000000                     171                =F'0'
000E20 000007D0                     172                =F'2000'
```